Figure 1: Refraction in a Washbasin

An Example Two-Day Unit – Two Days in the Life of a Scientist

This unit tells a story – a story of a scientist starting in well-trodden, comfortable territory with a straightforward application of standard skills – but then encountering an unexpected and possibly unsettling surprise and checking that surprise with a simple experiment. As with all chapters in the life of a scientist, we begin with observations in the real world. The night before the first day students do the experiment shown in Figure 1 at home looking in a washbasin and observing how water distorts the appearance of an object underwater.

This unit is designed for a math class or a programming class when students are learning how to minimize a function. It is also a good unit for a STEM workshop. It can be used at many levels and with different technology support.

This document is fairly long because it can be used in many different courses at many different levels and with many different teaching styles.

Evening Before Day 1:

The centerpiece of this assignment is the experiment shown in Figure 1. Students should look at an object at the bottom of a washbasin, first with the washbasin empty and then with it full. They should observe that when the washbasin is filled with water the object appears to be higher than it actually is. Students will also need the following for their work in this unit. Depending on your course and style you may have already covered these, may need to introduce or review them, or may (my own preference) have them look them up on the Internet before coming to class for Day 1.

- Finding the minimum of a function of a single variable on a closed interval. Depending on your class this may be the skill whose use and application you're teaching at this

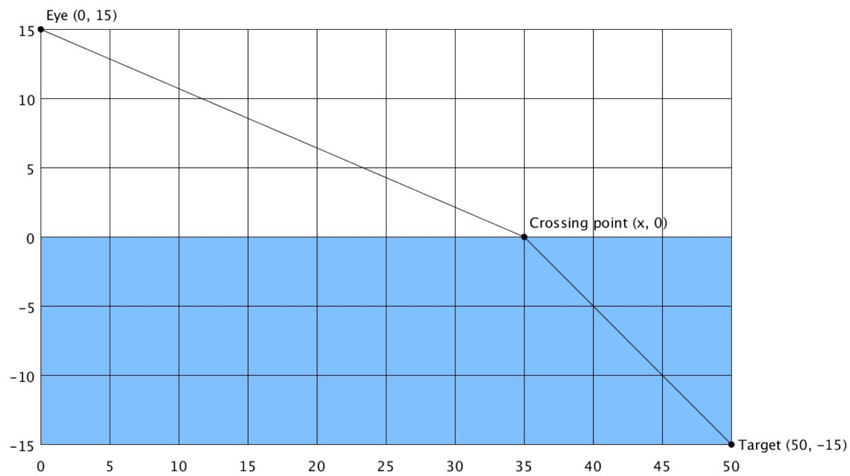

Figure 2: A Possible Path

point in the course. It may be done using Calculus or with technology – anything from a graphing calculator or programming to a computer algebra system.

- Fermat's Principle. We use the simplest form – light traveling between two points follows the fastest route.
- The speed of light in air and in water.

Day 1:

I usually have my students work in groups. Your style may be different. This first day can probably be done individually with some plenary discussion at the end. The second day I strongly recommend group work with plenary discussion at the end.

Figure 2 shows a straightforward application of Fermat's Principle that can be used to understand the phenomena we see when we look at the drain in the bottom of a washbasin. You may want to have all your students use the same example or have each student (or group) use their own example. For example, if some of your students are fishermen they might want to use an example with a fly fisherman up to her waist in water.

In Figure 2 an eye located 15 cm above the surface of the water in an aquarium is looking at a target 15 cm below the surface of the water at the other end of the 50 cm long aquarium. Using the coordinate system shown in the figure the location of the eye is $(0, 15)$ and the location of the target is $(50, -15)$. The x-axis runs along the surface of the water. The shortest possible path from the target to the eye would be a straight line but this is not the fastest possible path because light travels faster (30.0 cm per nanosecond) in air than it does in water (22.5 centimeters per nanosecond). In Figure 2 we see one possible

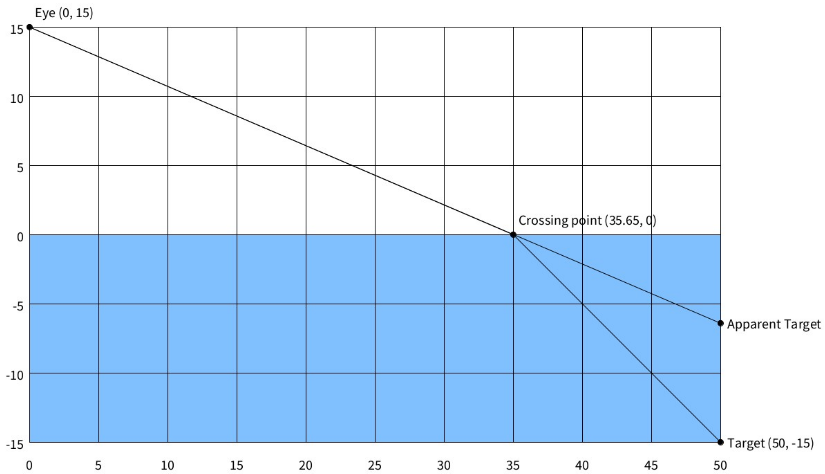

Figure 3: Apparent Path from One Eye

path light might follow. We use the letter x to denote the x-coordinate of the crossing point because we don't know where the best possible crossing point is. The function

$$f(x) = \frac{\sqrt{x^2 + 15^2}}{30.0} + \frac{\sqrt{(50 - x)^2 + 15^2}}{22.5},$$

computes the total travel time in nanoseconds for a light ray following the path shown in Figure 2. To determine the actual path followed by our light rays we must find the value of x that minimizes the function $f(x)$. For this example, the fastest path crosses the water's surface at $x \approx 35.65$. See Figure 3. Here are the three questions I pose for my students on this first day in class.

Question 1

Find the point $(x, 0)$ *where light traveling from the target at the point* $(50, -15)$ *to the eye at the point* $(0, 15)$.

Question 2

Find the apparent position of the target in the aquarium shown in Figure 2. How deep does it appear to be? Remember you have two eyes.

Question 3

Suppose that a fish that is 5 cm long and 2 cm tall is 15 cm below the surface of the water at the end of our 50 cm aquarium. How long and how tall does it appear to be to our usual observer?

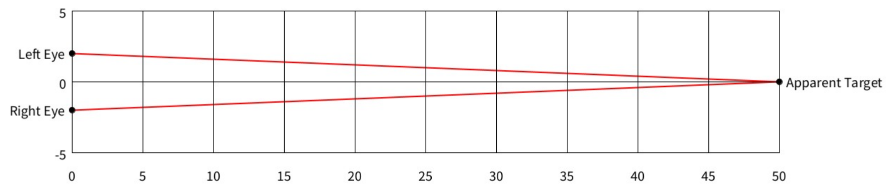

Figure 4: Two Apparent Paths from Above

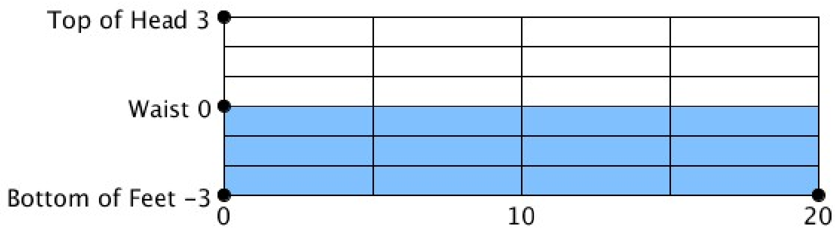

Figure 5: What a Fish Sees

We want to reach closure on the three questions above in a plenary discussion at the end of class on Day 1. Be sure to discuss the following.

When you look at an underwater target, you use two eyes and your brain determines the apparent location of the target using the information received from both eyes. For each eye alone we look at Figure 3 but we also need Figure 4, which shows a view looking down on the aquarium from directly overhead. Think of your nose pointing at the target with one eye on each side of your nose looking at the target. The two red lines show the two apparent paths from above. Since we are looking directly down from above we don't see the bends in the apparent paths. The two apparent paths determined by the two eyes cross directly above the target. Thus, your brain thinks the x-coordinate of the target is unchanged from its actual x-coordinate.

If you or any of your students are bow fishermen you or they won't be surprised by this analysis. Bow fishermen know that they must aim below a fish's apparent position. Day 2 will produce some real surprises for all with the possible exception of fly fishermen and scuba divers. As preparation tell your class that tomorrow we plan to investigate what a fish sees.

Day 2:

I use Figure 5 with feet as the unit of length. It shows a six foot tall fly fisherman up to his waist in a three foot deep pond looking at a fish on the bottom 20 feet away. I ask the students a single question:

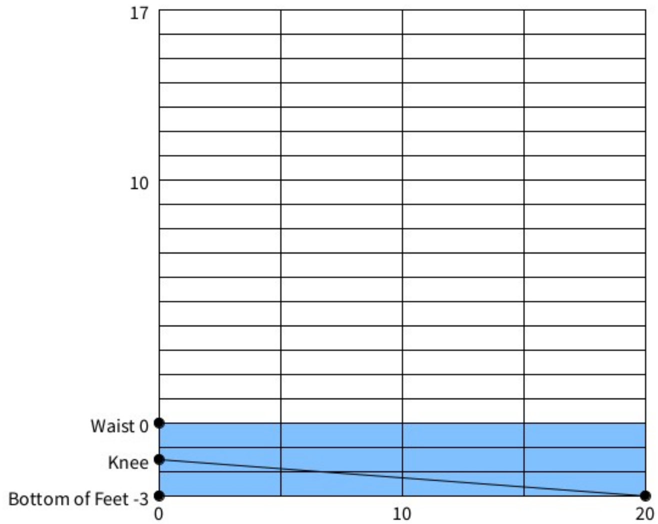

Figure 6: The Fish Looking at the Fisherman's Knee

Question 4

What does the fish see?

I expect students to produce Figures 7, 8 and 6 and note the following:

- The fish sees the part of the fisherman that is below the water's surface unchanged because light traveling from any point on this part to the fish is traveling through one medium, water, and the fastest path is the shortest path, a straight line. See Figure 6.
- The top of the fisherman's head appears to be far, far above its actual position. See Figure 7.
- A point on the fisherman's waist just a tiny distance above the water's surface also appears far, far above its actual position. The fastest path from this point to the fish's eye skims just above the water's surface for a long time before entering the water. See Figure 8.

The net, surprising, perhaps unsettling, result of all this is the the fish sees the fisherman cut in half with his head and torso floating far above the water's surface. Anyone living the life of a scientist often encounters surprises that run counter to expectations. We

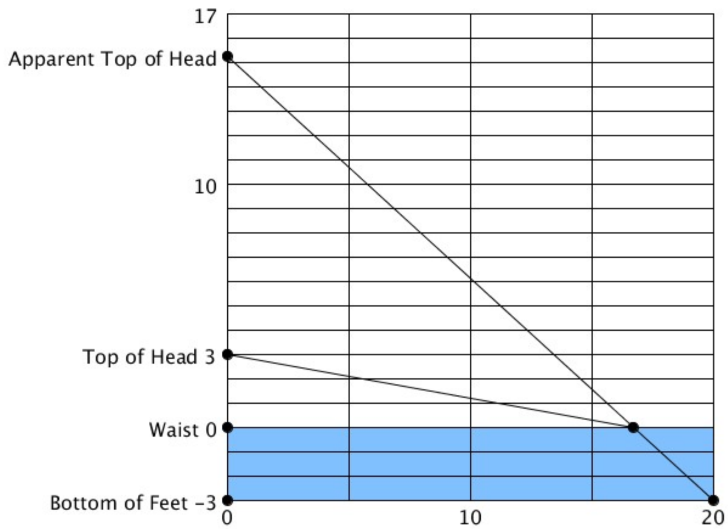

Figure 7: The Fish Looking at the Top of the Fisherman's Head

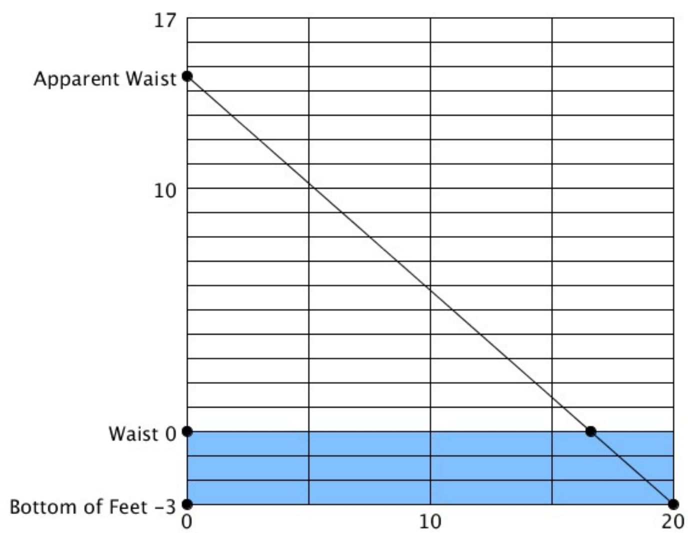

Figure 8: The Fish Looking at the Fisherman's Waist

must verify unexpected results by experimentation and observation. Figure 9 shows an experiment I did on my kitchen counter and Figure 10 shows the result of this experiment. The photograph on the left side of Figure 10 was taken from above the water at the end opposite the yardstick. The photograph on the right side of Figure 10 was taken using an underwater camera at the bottom of the aquarium at the end opposite the yardstick on the right side. You should do this experiment in class at the end of Day 2.

Notice that as predicted by our analysis the yardstick appears to the fish to be cut in half at the water's surface with the upper part of the yardstick floating high above the water. There is another surprise. The region between the underwater portion of the yardstick and the above water part of the yardstick apparently floating high above the water is filled by a reflection of the underwater part of the scene.

As always we are left with more questions and we and our students have lots of everyday observation to draw upon. Many of our questions center around what happens when a light ray encounters a surface separating two mediums, like air and water. Sometimes it bounces or is reflected and sometimes it passes through. We've all observed reflections in the surface of a puddle. Sometimes "part" of the light is reflected and sometimes another "part" passes through. Most of us have experience with Polaroid$^{\mathrm{TM}}$ glasses and have noticed how they can be used to remove reflections. Maybe polarization has something to do with why some light passes through a surface and other light is reflected.

Our two-day unit has two punchlines:

- Surprise, verification and a sense of wonder or amazement.
- More questions – perhaps even a future career.

Figure 9: Kitchen Counter Experiment

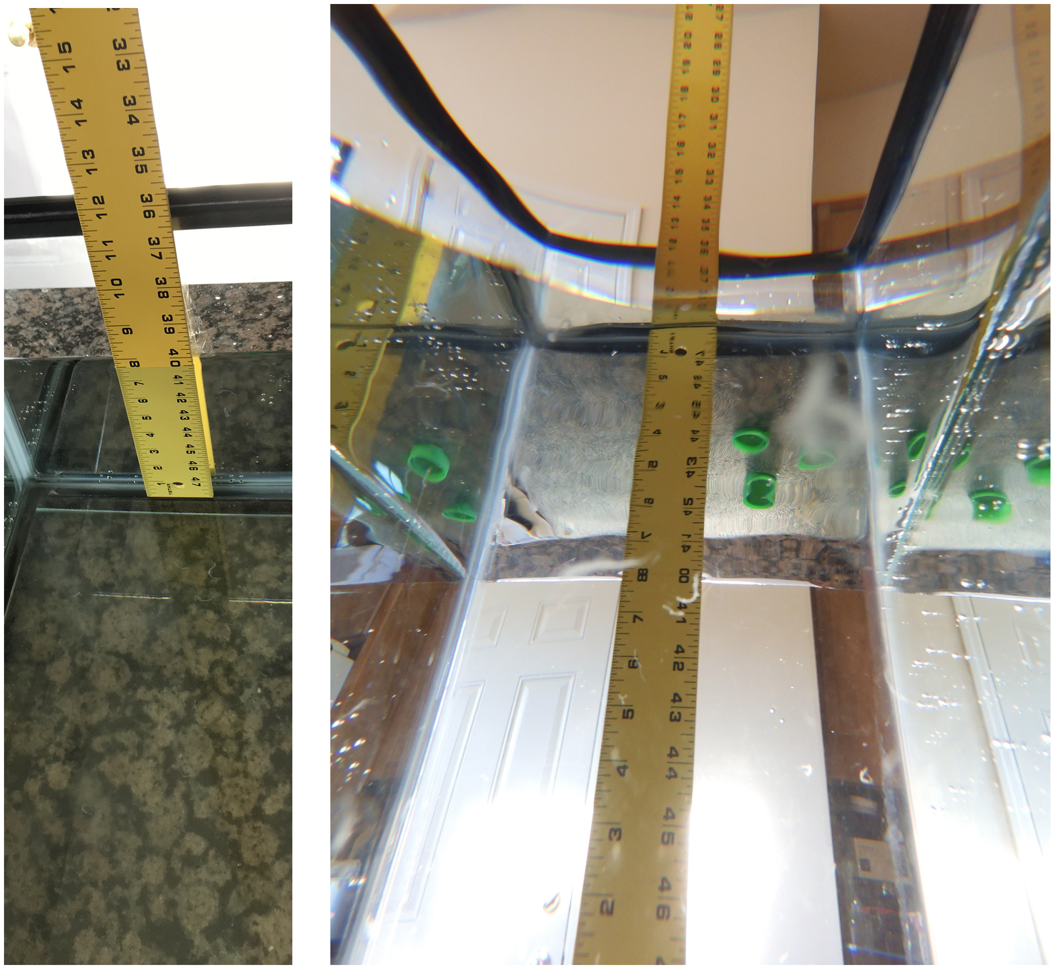

Figure 10: Verifying the Surprise View Seen by a Fish