Modeling, Geometry and Reflections in a Bathroom Mirror

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Figure 1. My reflection in a mirror

You probably look at your reflection in a mirror many times during the day. See, for example, Figure 1, which shows my own reflection in a bathroom mirror. Ubiquitous and seemingly simple reflections like these are a wonderful setting for learning about mathematical modeling and its power. All we need is a camera (cellphone cameras work great), some high school geometry and a mirror. We will learn how scientists and mathematicians use models to understand and make predictions about our world and how they use experiments and observation both to build models and to test their validity and usefulness. Figure 2 below is the key to using geometry to understand the reflections we see in mirrors.

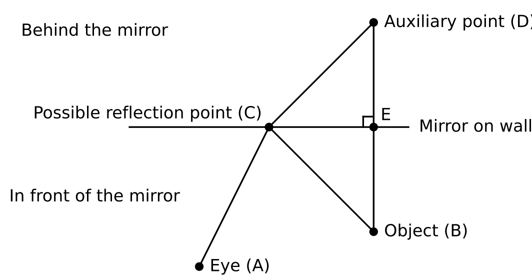


Figure 2: Possible reflection point

In this figure we are looking downward from above at a scene like Figure 1. The horizontal line, helpfully labeled "Mirror on wall," represents the mirror on the wall. The portion of the figure below this horizontal line represents the part of the scene in front of the mirror. Notice one dot, labeled "Eye (A)," representing an eye and a second dot, labeled, "Object (B)," representing one point that the eye might be looking at (or, more precisely, whose reflection in the mirror the eye might be looking at). When you look at your reflection in a mirror, it looks like your reflection is behind the mirror and the wall on which the mirror is mounted. This area behind the mirror is represented in Figure 2 by the portion of the figure above the line representing the mirror.

The eye and object are both in front of the mirror. A light ray is traveling from the object to the mirror and bounces off the mirror at a "bounce point" or "reflection point." One possible reflection point is marked by C. We denote the line segment between the points A and C by $\overline{AC}$ and its length by $|\overline{AC}|$. Fermat's principle[1] implies that the light ray will "choose" the reflection point C that minimizes the total distance it must travel, the length of the line $\overline{AC}$ plus the length of the line $\overline{CB}$, or $|\overline{AC}| + |\overline{CB}|$.

We have added an auxiliary point marked by D. It is directly behind the mirror, the same distance from the mirror as the object B and directly opposite the object B. The line segment $\overline{BD}$ is perpendicular to the mirror and the line segments $\overline{DE}$ and $\overline{BE}$ have the same length. The triangles ΔCDE and ΔCBE are congruent – that is, they have exactly the same sides and angles. Thus, $|\overline{AC}| + |\overline{CB}| = |\overline{AC}| + |\overline{CD}|$.

Since the shortest distance between two points is a straight line we can minimize $|\overline{AC}| + |\overline{CD}|$ by moving C so that it lies on the straight line between the points A and D as shown in Figure 3 below. The eye at the point A thinks the object is along this straight line through the two points A and D. We will see in a moment that with two eyes, a person thinks the object is at the point D. So we call this point the "apparent point."

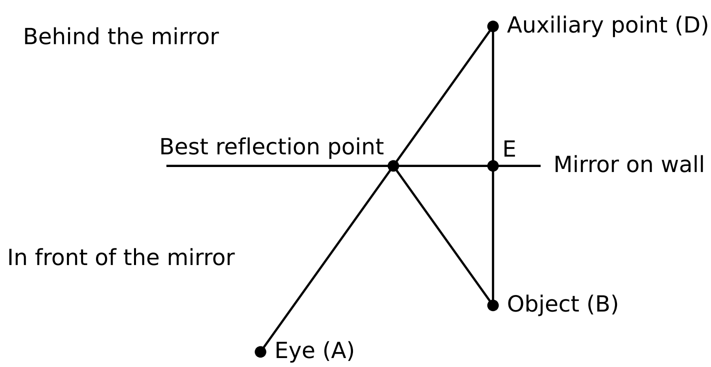


Figure 3: Best reflection point

[1]In its simplest form Fermat's Principle states that light traveling between two points takes the fastest path.

Figure 4 below shows why a person using binocular vision thinks the object is at the apparent point. The analysis of a light ray traveling from the object B to the mirror and then to the eye F is similar to the analysis of a light ray traveling from the object B to the mirror and then to the eye A.

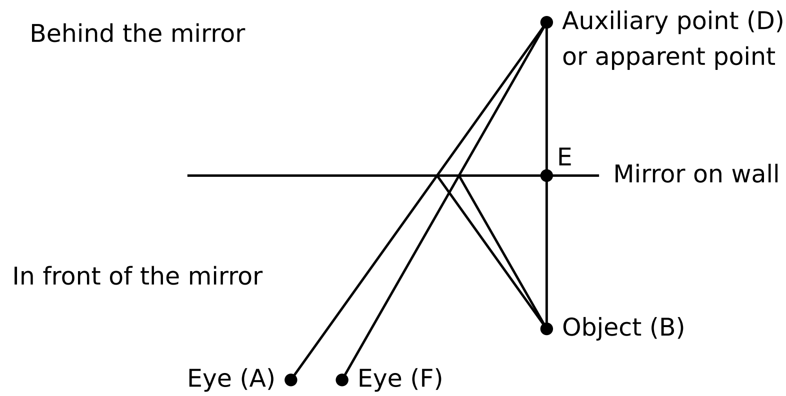
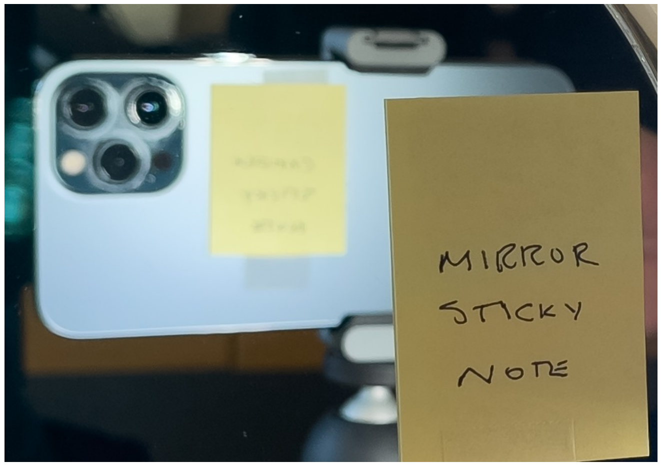
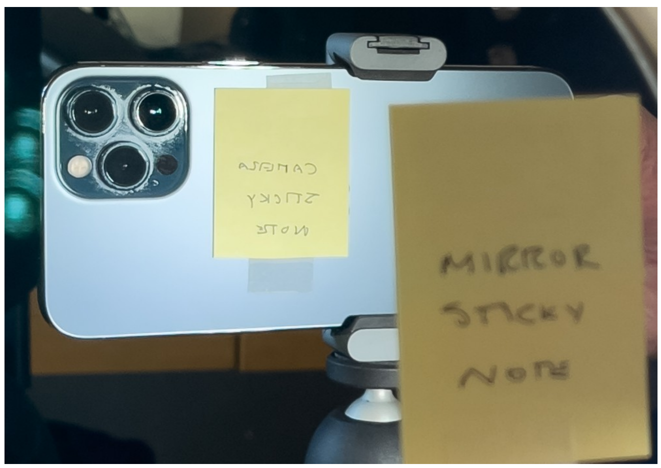


Figure 4: Binocular vision with two eyes

The two photographs in Figure 5 show one example of an experiment you may be able to do with one eye or with a camera. These photographs were made with a cellphone camera. You can try to do the same experiment with one eye. In the top photograph the camera is focused on the sticky note taped to the surface of the mirror. Notice that sticky note on the mirror is sharp but the sticky note taped to the camera is out-of-focus in the reflected image because the reflected image is much further away from the camera than the surface of the mirror. In the bottom photograph the camera is focused on the reflection of the sticky note taped to the camera. Notice that reflected sticky note on the camera is sharp but the sticky note taped to the surface of the mirror is out-of-focus because the surface of the mirror is much closer than the reflected image of the camera. With many cellphone cameras you can choose which point to focus on by tapping the chosen point on the cellphone screen. You can try the same experiment looking with one eye closed at your reflected image in a flat mirror and consciously switching the focus of your eye.



Focusing on the mirror sticky note



Focusing on the cellphone camera sticky note

Figure 5

If you Google “Why do mirrors flip left to right?” you will get a large number of hits. You may have asked this question yourself. Our work above, confirmed with a quick experiment, can answer this question. Print the words “FUNNY TEE SHIRT” in large dark letters on a thin sheet of paper, hold this paper on your shirt and look at yourself in the mirror. Sure enough, the words seem to be reversed left to right. The apparent image of the words has been moved straight back behind the mirror so that you’re looking at the exact same words from behind. Check this analysis by extending the paper with the words in front of you, so that you can see the words through the paper – Voila!!

Our bathroom experimentation with just our eyes and, optionally a camera like a cellphone camera, has followed the same storyline followed by the most famous Nobel-winning scientists in the most expensive laboratories.

- We began with observation and experimentation. We observed our reflection in a mirror and maybe experimented, for example, by waving one arm to see which reflected arm waved back.
- Building on the work of others, in this case elementary geometry and Fermat's Principle, we built and analyzed a mental model to explain our observations. Our mental model made some predictions.
- We checked those predictions back in our bathroom laboratory – the real world. See Figure 5 and your experiment with the words "FUNNY TEE SHIRT" on a thin piece of paper. This step is called "validation." We compared our results – the predictions of our model – not with answers in the back of the book but with the real world.