

Differential Equations: One Course in a Lifetime of Modeling



Frank Wattenberg

Frank.Wattenberg@mac.com

<https://justaddmath.org/simiode-expo-2023/>

U.S. Military Academy (emeritus)

Modeling is essential for understanding and bettering our lives
EXPLANATORY vs DESCRIPTIVE -- GOALS and VALUES

Three Points

- Ordinary and Partial Differential Equations are one paradigm in a growing modeling toolkit or repertoire. Use the whole toolkit. Qualitative methods.
- The primary purpose of modeling or the scientific method is building understanding and giving us the power to change our world
- Modeling is lifetime, everyday, everywhere (around the house below)



The Mid-Hudson Bridge

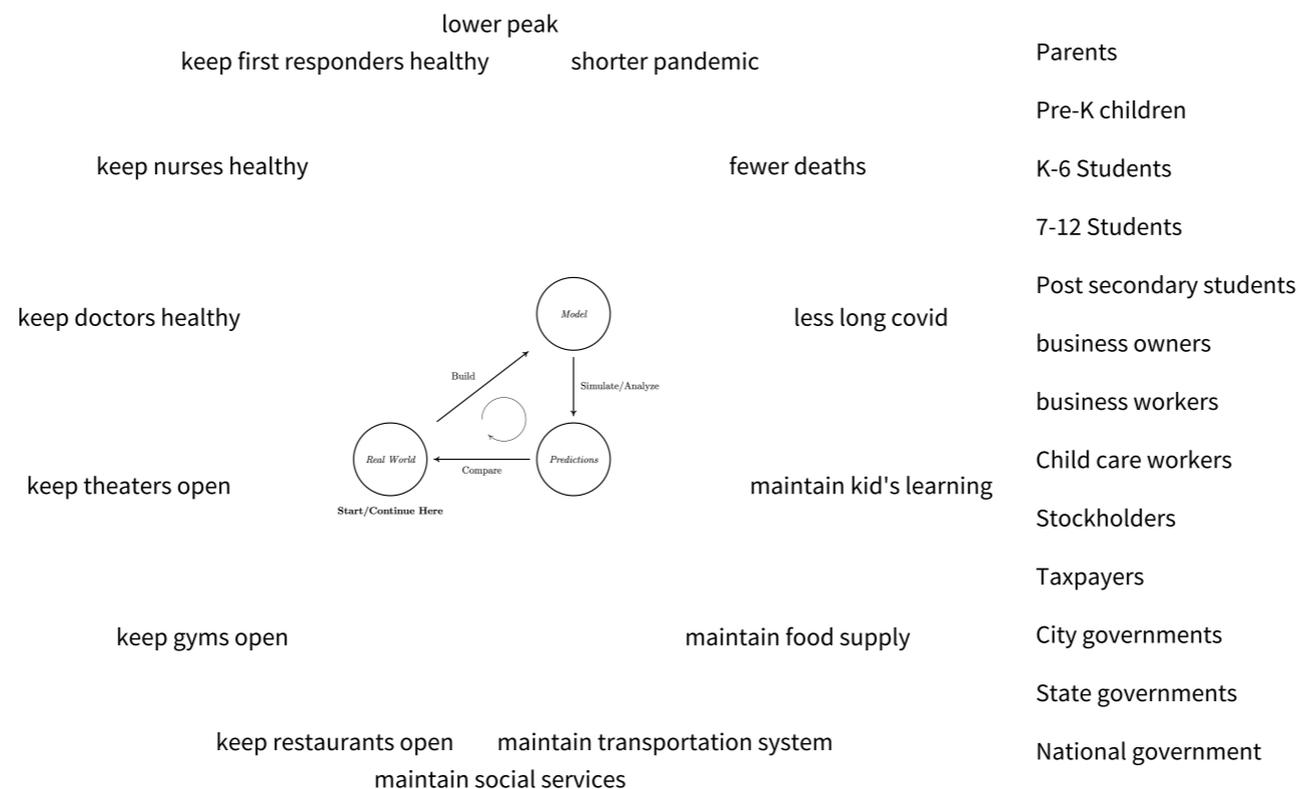


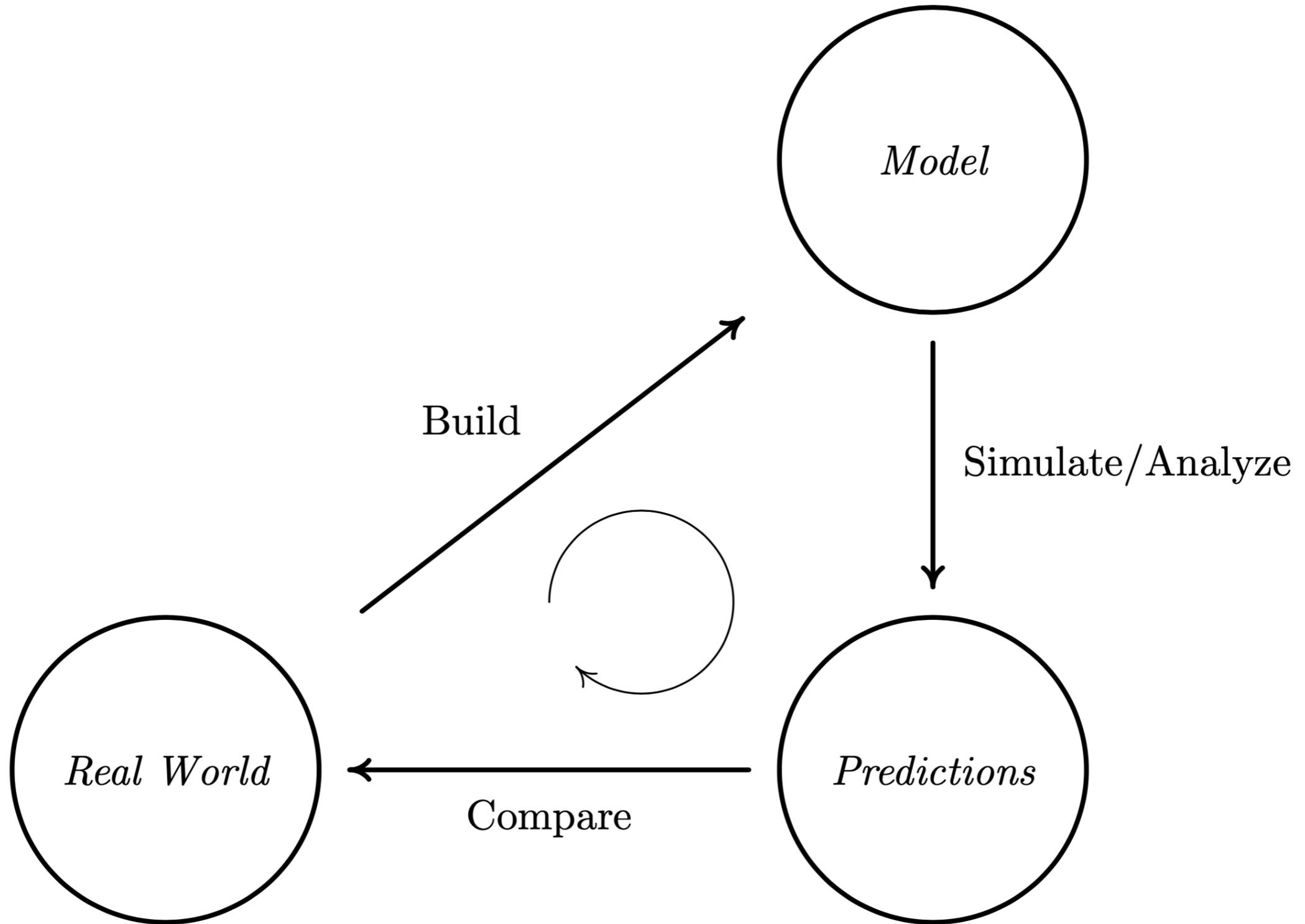
Joseph Bertolozzi

Open Discussion

Today (this room) after five minute break

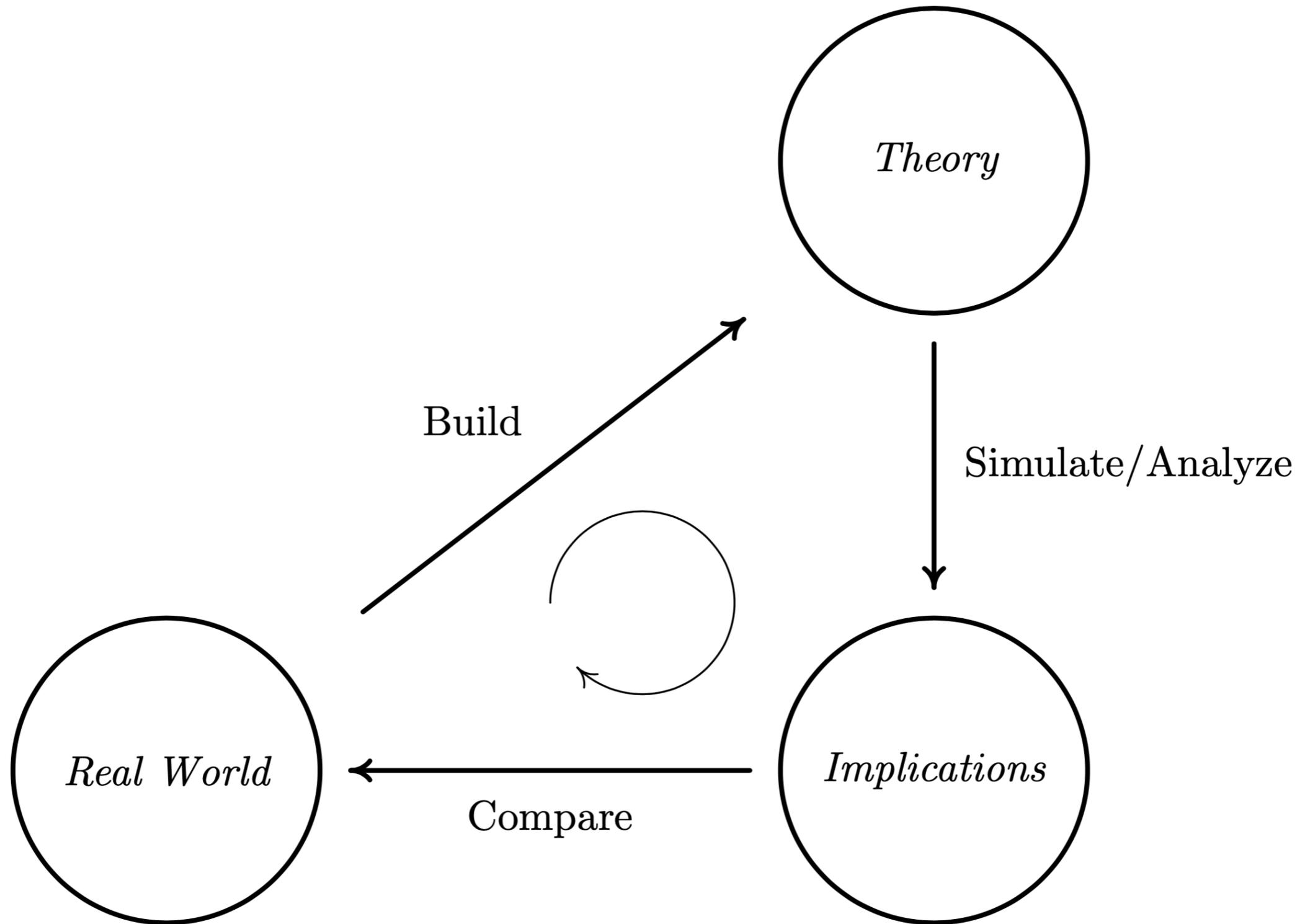
- Experiments and observations around the house.
- Experiences with ChatGPT, text-based image search and generation.
- Ideas for Modeling for High Stakes Decisions.





Start/Continue Here

Figure 1: The modeling cycle



Start/Continue Here

Figure 2: The Scientific Method

lower peak
keep first responders healthy shorter pandemic

Parents

Pre-K children

keep nurses healthy

fewer deaths

K-6 Students

7-12 Students

Post secondary students

business owners

business workers

Child care workers

Stockholders

Taxpayers

City governments

State governments

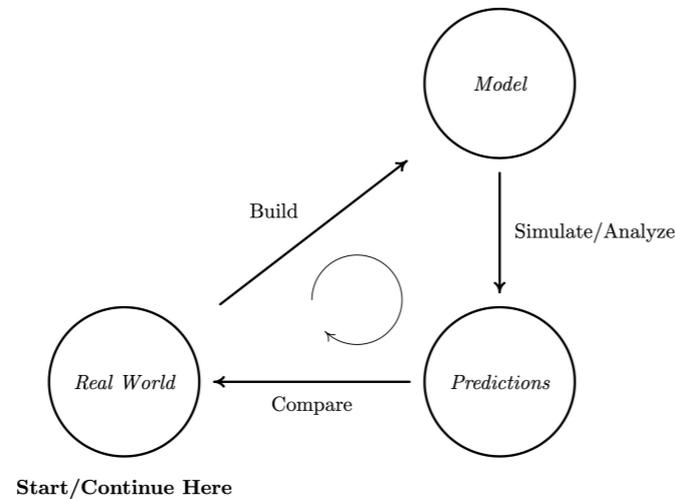
National government

keep doctors healthy

less long covid

keep theaters open

maintain kid's learning



keep gyms open

maintain food supply

keep restaurants open maintain transportation system
maintain social services

Modeling, Goals and Stakeholders (e.g. COVID-19)

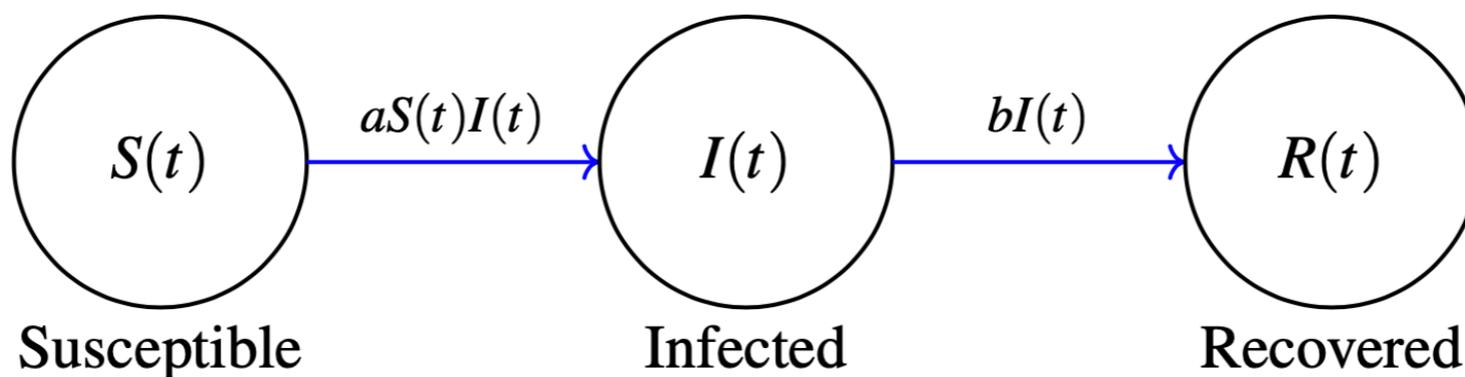
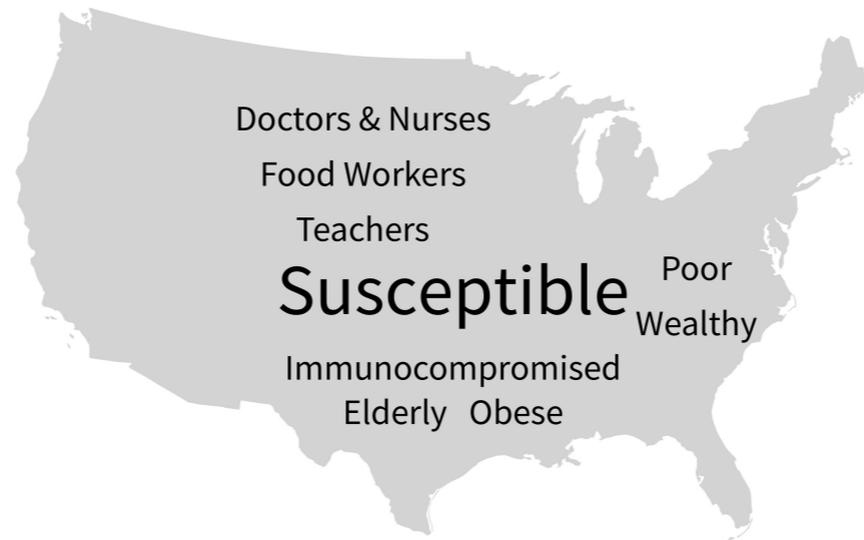


Figure 7.1: Susceptible, recovered, and infected (SIR) compartmental model of an epidemic.

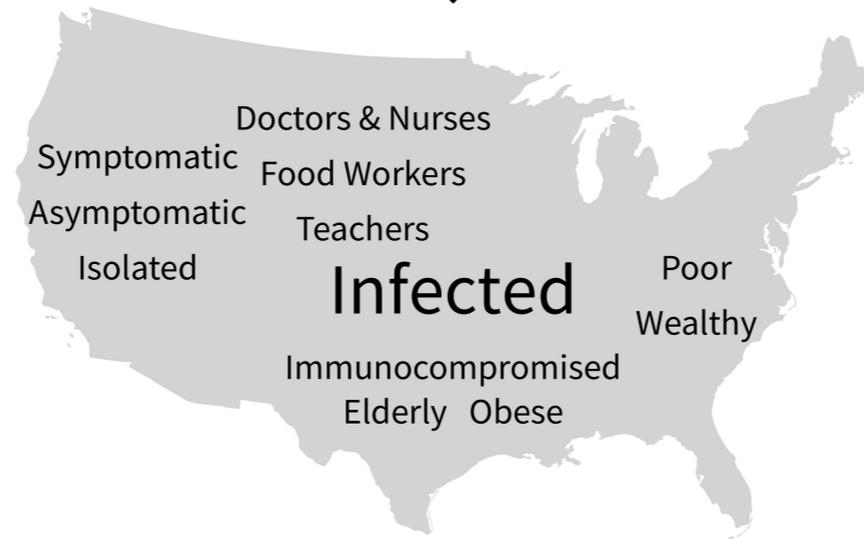
As in any compartment model, we must specify the rate at which the quantities of interest (in this case, people) move between the compartments. In the classic SIR model, susceptible and infected people interact at a rate that is proportional to the product SI . The reasoning is that, for any fixed value of S , if the value of I is doubled then the number of interactions between susceptible and infected people should double, and a similar result should hold if I is fixed but S is doubled; the quantity SI captures this observation. Moreover, we suppose in the model that each such interaction carries a fixed risk of the susceptible person becoming infected and moving from the S to the I compartment. This is captured by the $aS(t)I(t)$ label above the arrow from the S compartment to the I compartment in Figure 7.1: the likelihood of infection is proportional to the number of interactions between the susceptible (S) and the infected (I) populations. The constant of proportionality a depends, for example, on the infectiousness of the disease. Movement from the I compartment to the R compartment is assumed to occur at a rate proportional to the number of infected people: all else being equal, if there are twice as many infected people then the number of people getting better per unit time should double.

With these observations we can posit the model

$$\begin{aligned}
 \dot{S} &= -aSI \\
 \dot{I} &= aSI - bI \\
 \dot{R} &= bI.
 \end{aligned}
 \tag{7.6}$$



$\downarrow a S(t) I(t)$



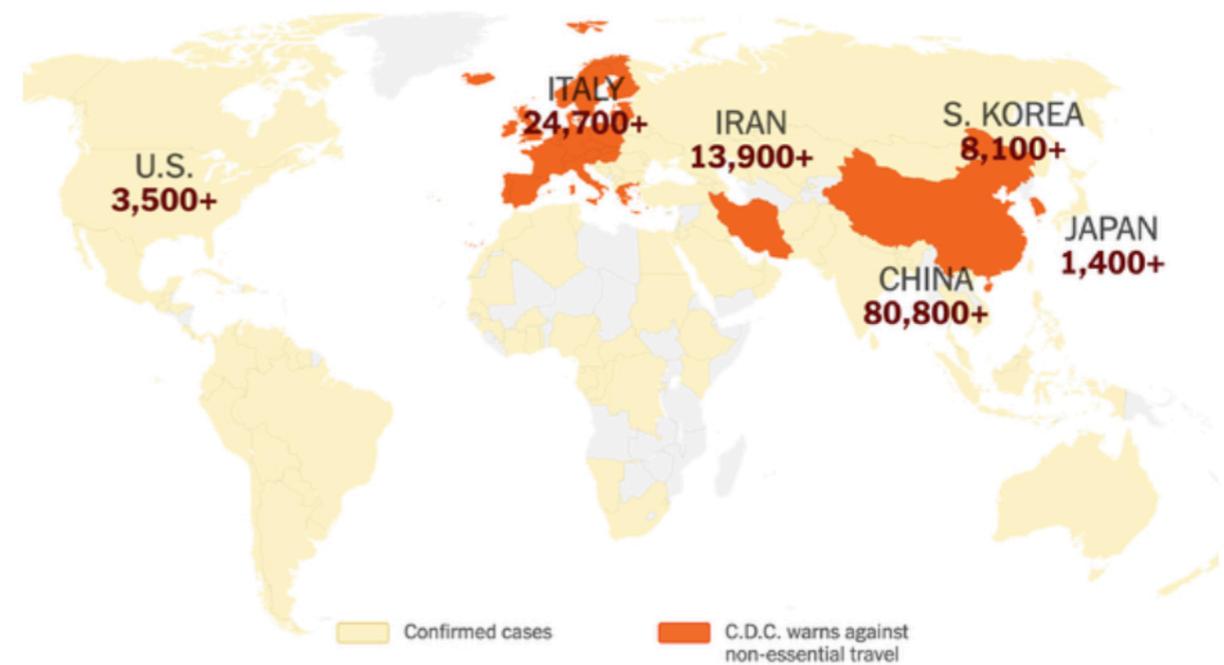
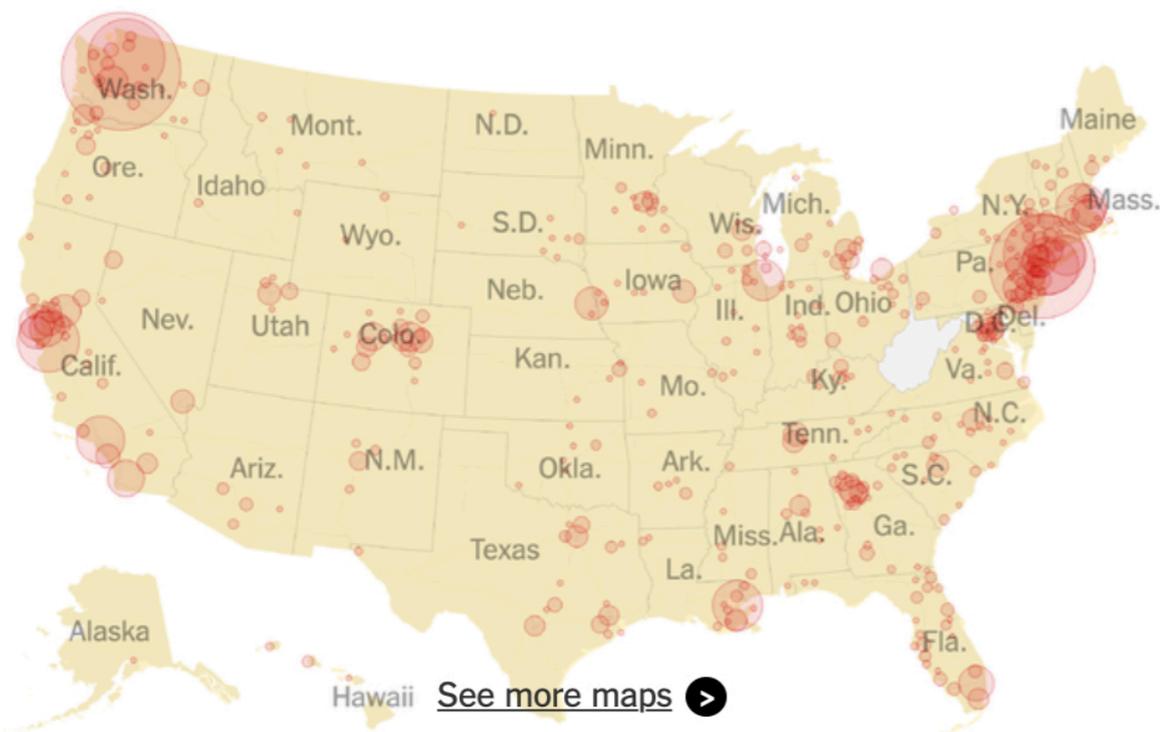
$\downarrow b I(t)$



Trump Wants U.S. 'Opened Up' by Easter, Despite Health Officials' Warnings

“You can’t just come in and say let’s close up the United States of America,” the president said, insisting again that he did not view the coronavirus as any more dangerous than the flu.

New York Times, March 24, 2020.



$$S' = -\alpha S \left(\frac{I}{S + I + R} \right)$$

$$I' = \alpha S \left(\frac{I}{S + I + R} \right) - \beta I - \gamma I$$

$$R' = \beta I$$

$$D' = \gamma I$$

```

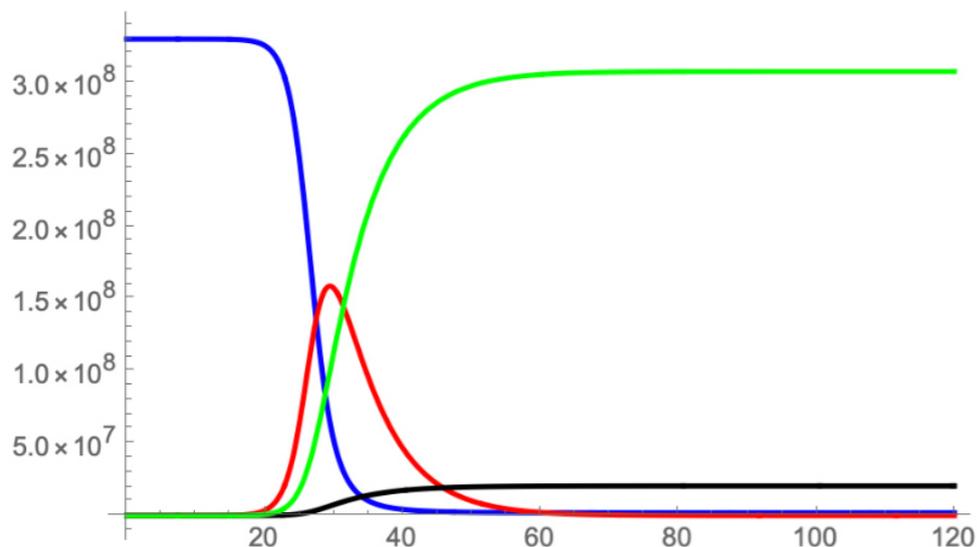
alpha = 0.80;
beta = 0.15;
gamma = 0.01;
endTime = 120;
Clear[susceptible, infected, recovered, deceased]
solution =
  NDSolve[ { susceptible'[t] == -alpha * susceptible[t] * (
    
$$\frac{\text{infected}[t]}{\text{infected}[t] + \text{susceptible}[t] + \text{recovered}[t]}$$

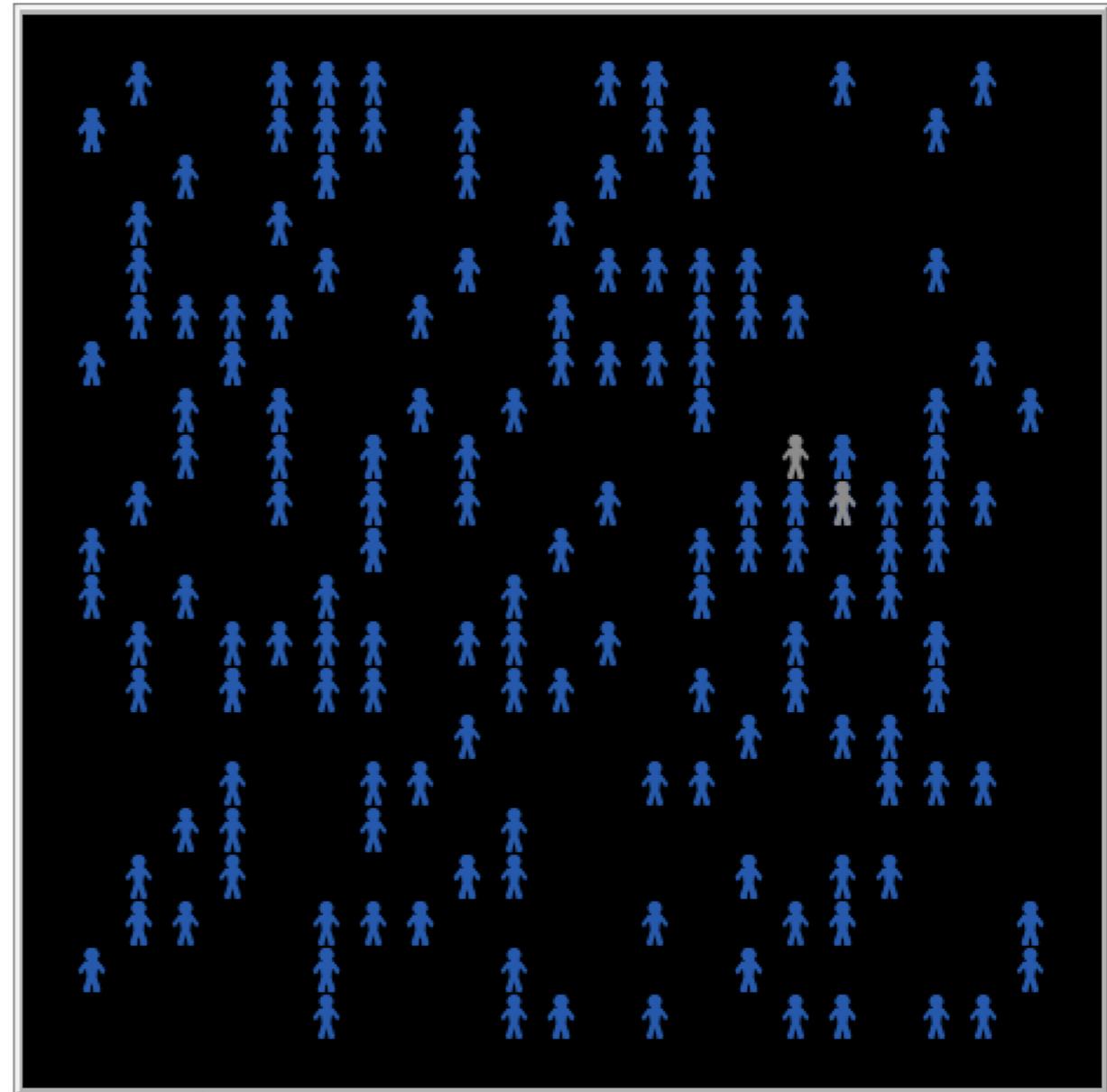
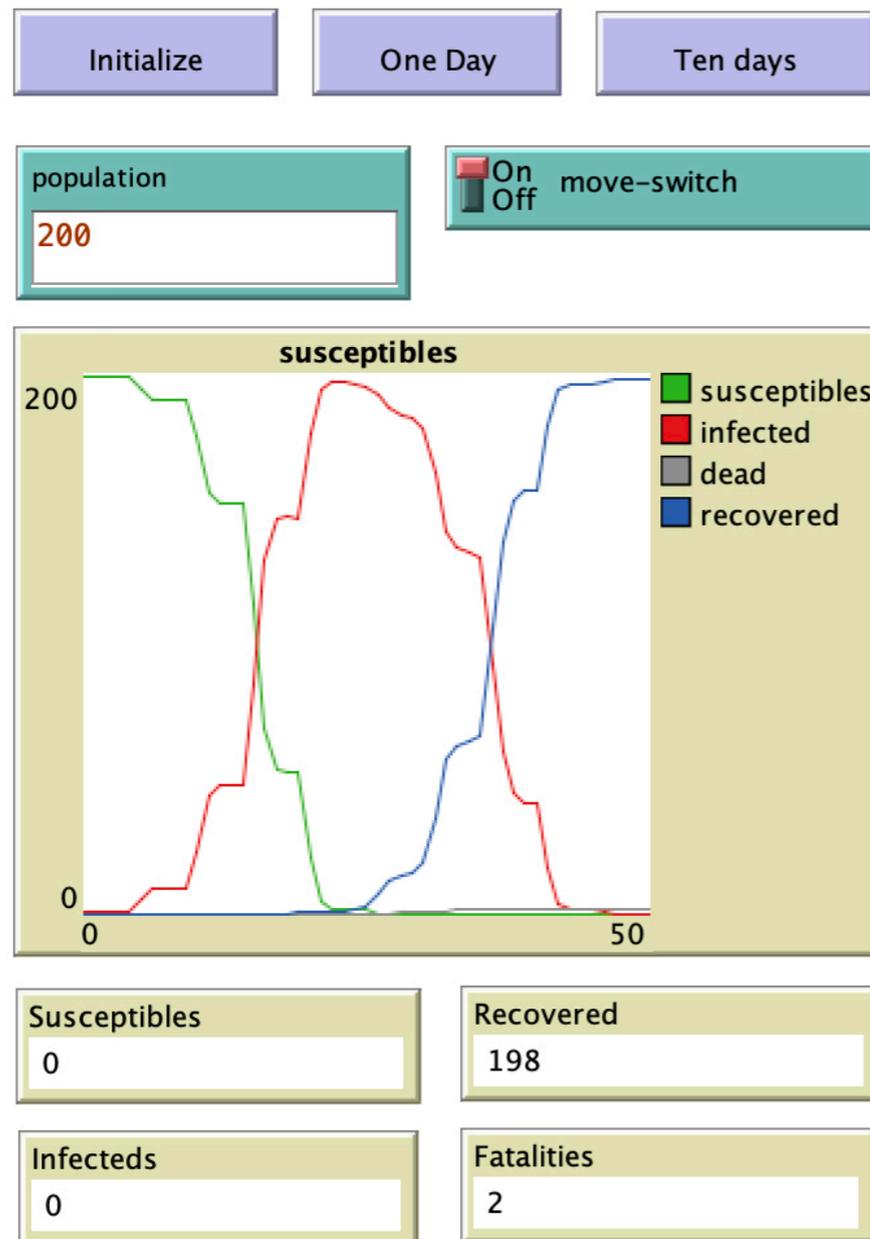
),
    infected'[t] == alpha * susceptible[t] * (
    
$$\frac{\text{infected}[t]}{\text{infected}[t] + \text{susceptible}[t] + \text{recovered}[t]}$$

) - (beta + gamma) * infected[t],
    recovered'[t] == beta * infected[t],
    deceased'[t] == gamma * infected[t],
    susceptible[0] == 330000000,
    infected[0] == 10,
    deceased[0] == 0,
    recovered[0] == 0 }, {susceptible[t], infected[t], recovered[t], deceased[t]}, {t, 0, endTime}];

susceptible[t_] = susceptible[t] /. solution[[1]];
infected[t_] = infected[t] /. solution[[1]];
recovered[t_] = recovered[t] /. solution[[1]];
deceased[t_] = deceased[t] /. solution[[1]];
Plot[{susceptible[t], infected[t], deceased[t], recovered[t]}, {t, 0, endTime},
  PlotStyle -> {{Blue, Thick}, {Red, Thick}, {Black, Thick}, {Green, Thick}}]
deceased[endTime]

```





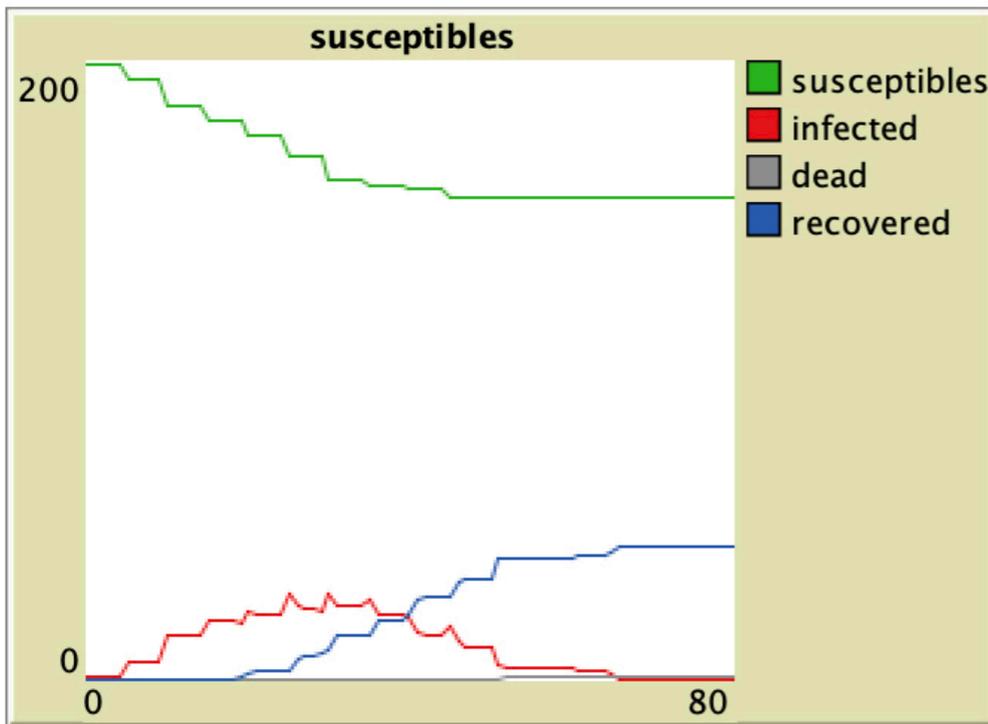
Agent-Based Modeling Object-Oriented Programming

NetLogo

Initialize One Day Ten days

population
200

On Off move-switch

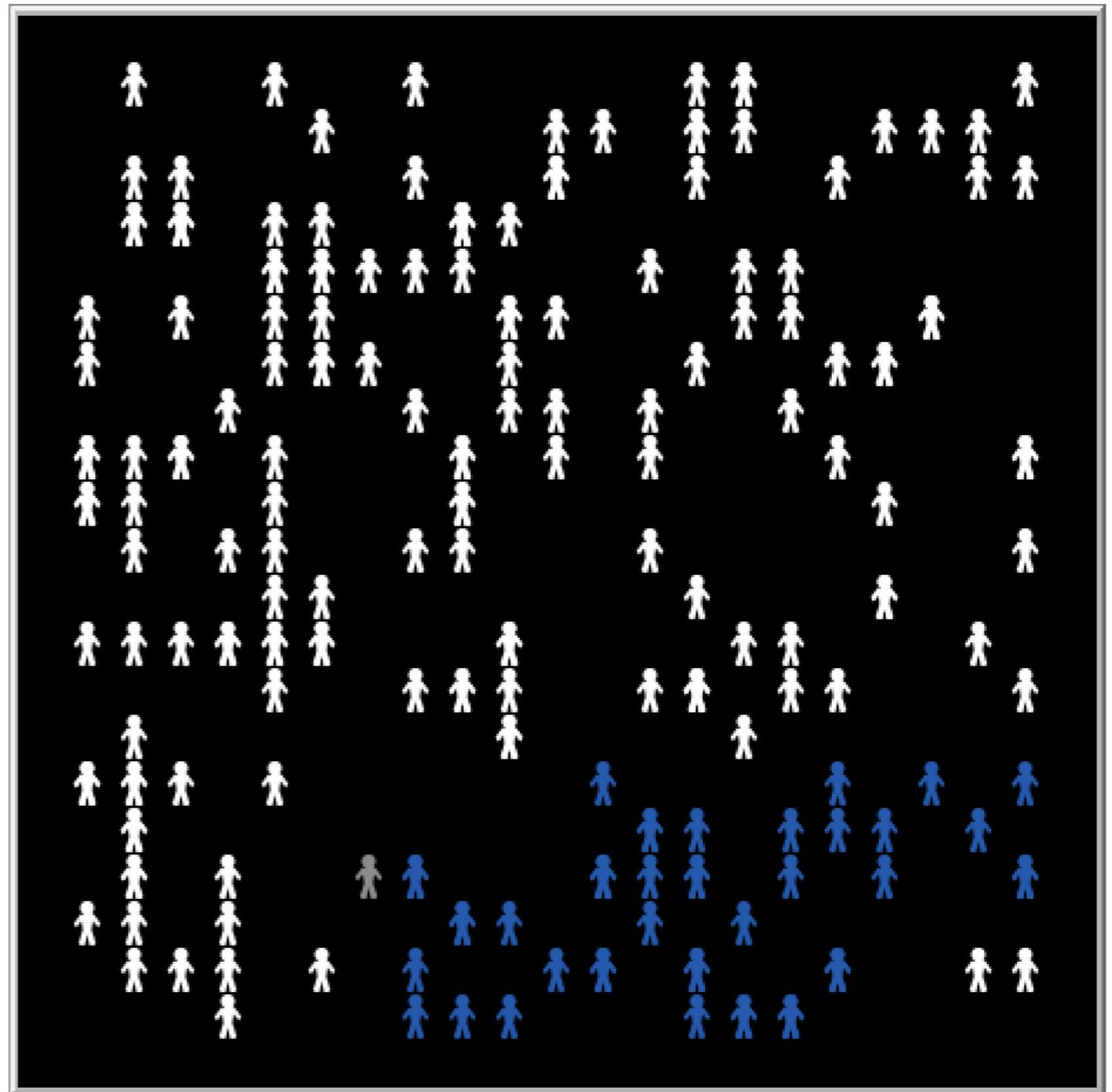


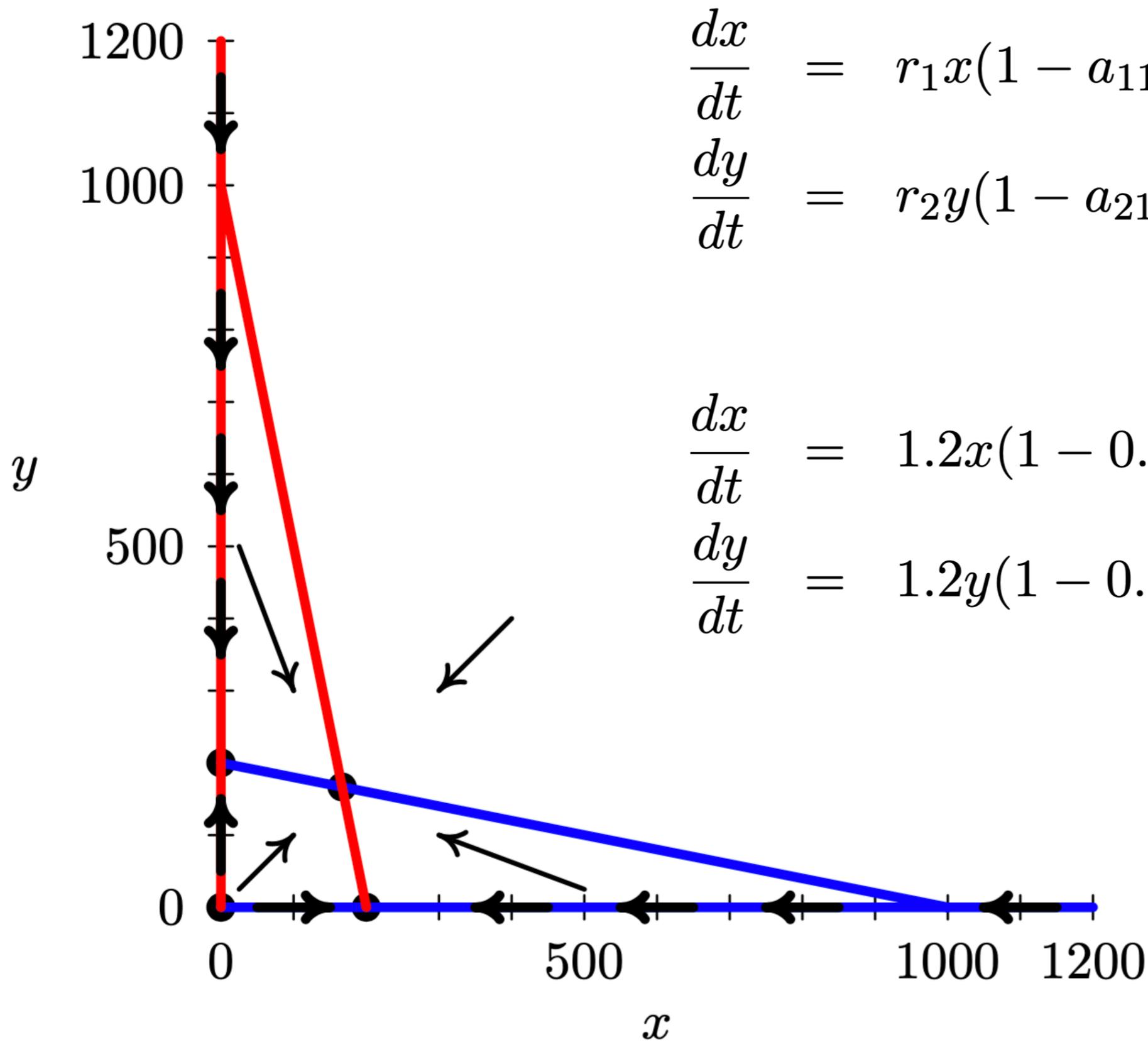
Susceptibles
156

Recovered
43

Infecteds
0

Fatalities
1



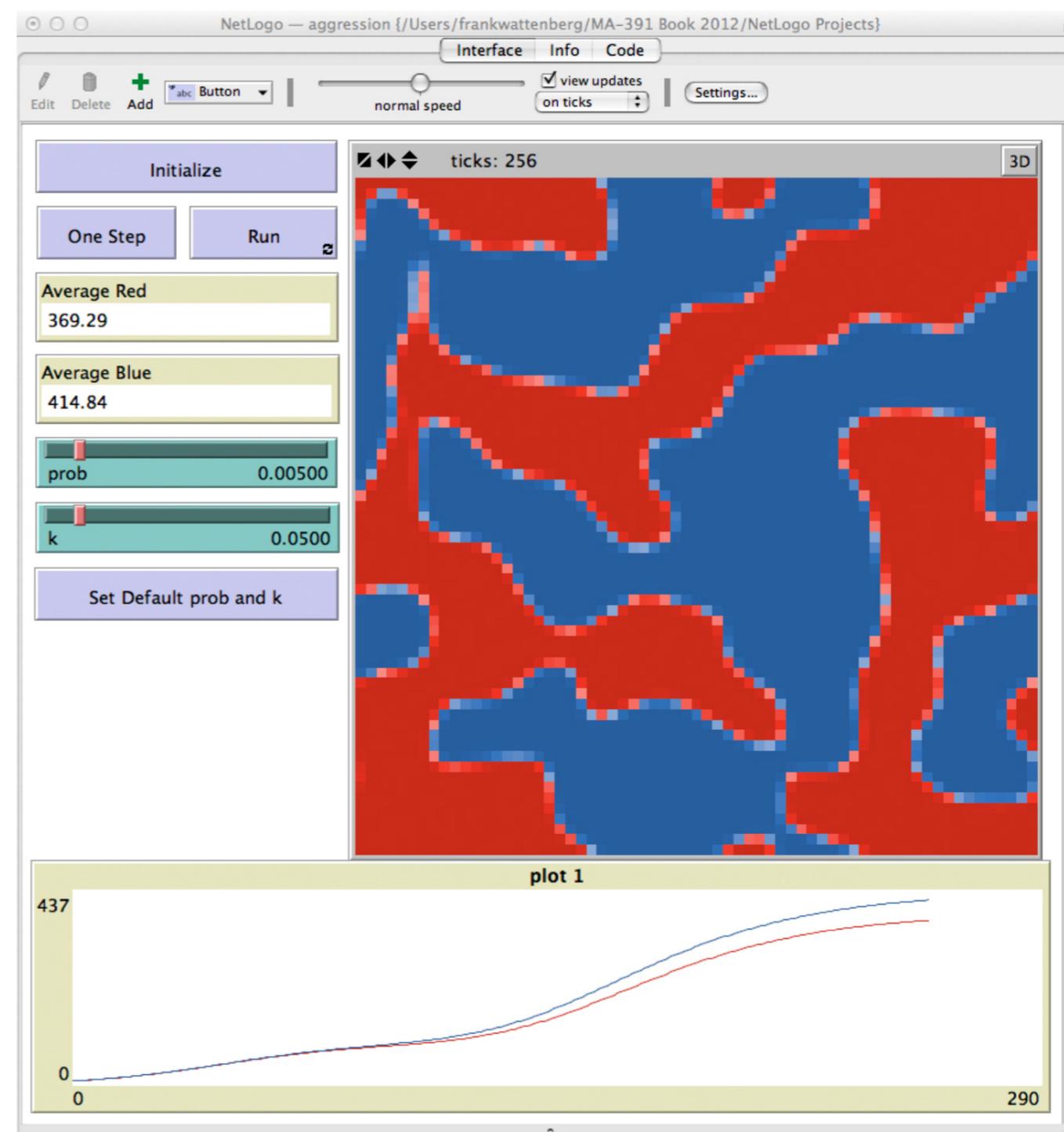
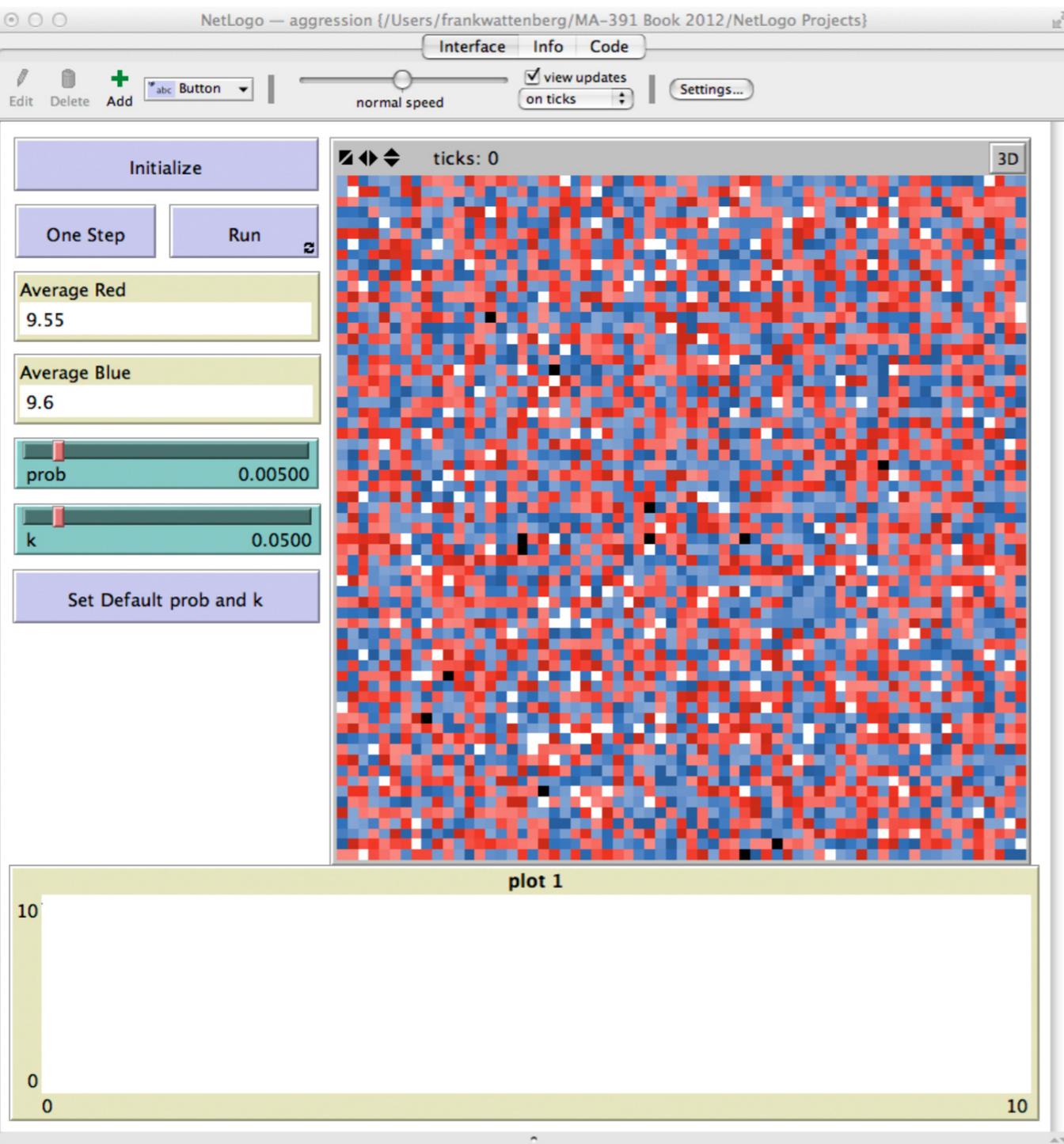


$$\frac{dx}{dt} = r_1 x (1 - a_{11} x - a_{12} y)$$

$$\frac{dy}{dt} = r_2 y (1 - a_{21} x - a_{22} y)$$

$$\frac{dx}{dt} = 1.2x(1 - 0.005x - 0.001y)$$

$$\frac{dy}{dt} = 1.2y(1 - 0.001x - 0.005y)$$



$$\frac{dx}{dt} = 1.2x(1 - 0.001x - 0.005y)$$

$$\frac{dy}{dt} = 1.2y(1 - 0.005x - 0.001y)$$

Discrete Time, Discrete Space and Agent-Based Models

Continuous Models

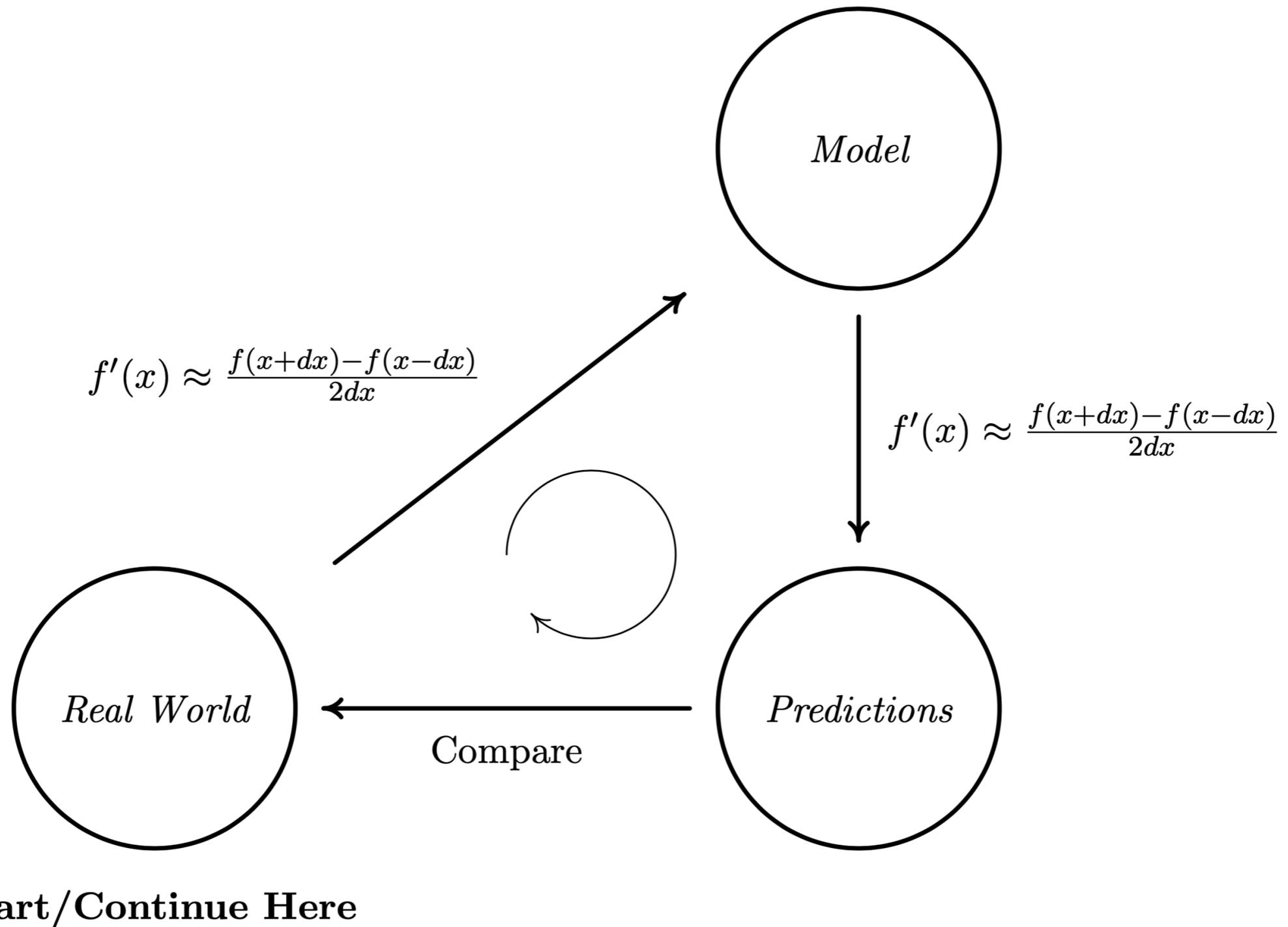


Figure 3: Dual Use

Quantum phenomena and quantum computing will challenge our modeling ability and provide new modeling paradigms.

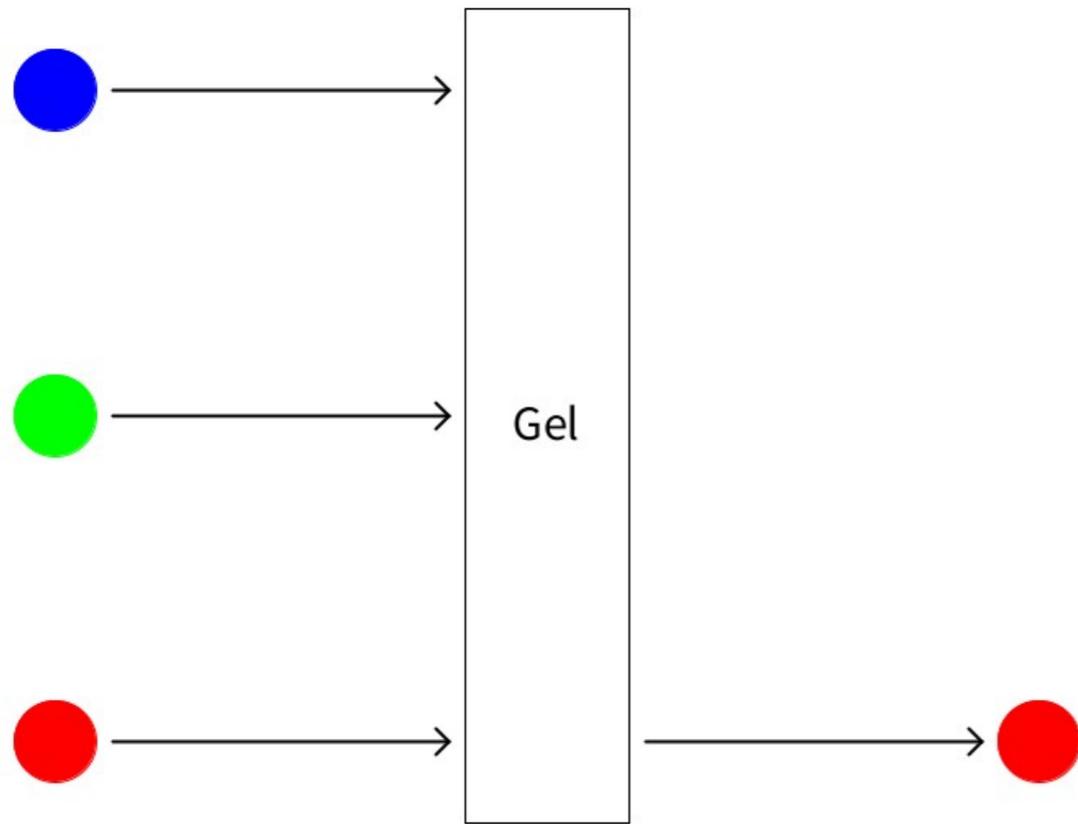
We need to understand these. Where is Isaac Asimov when we need him?

Superposition and Quantum Entanglement.

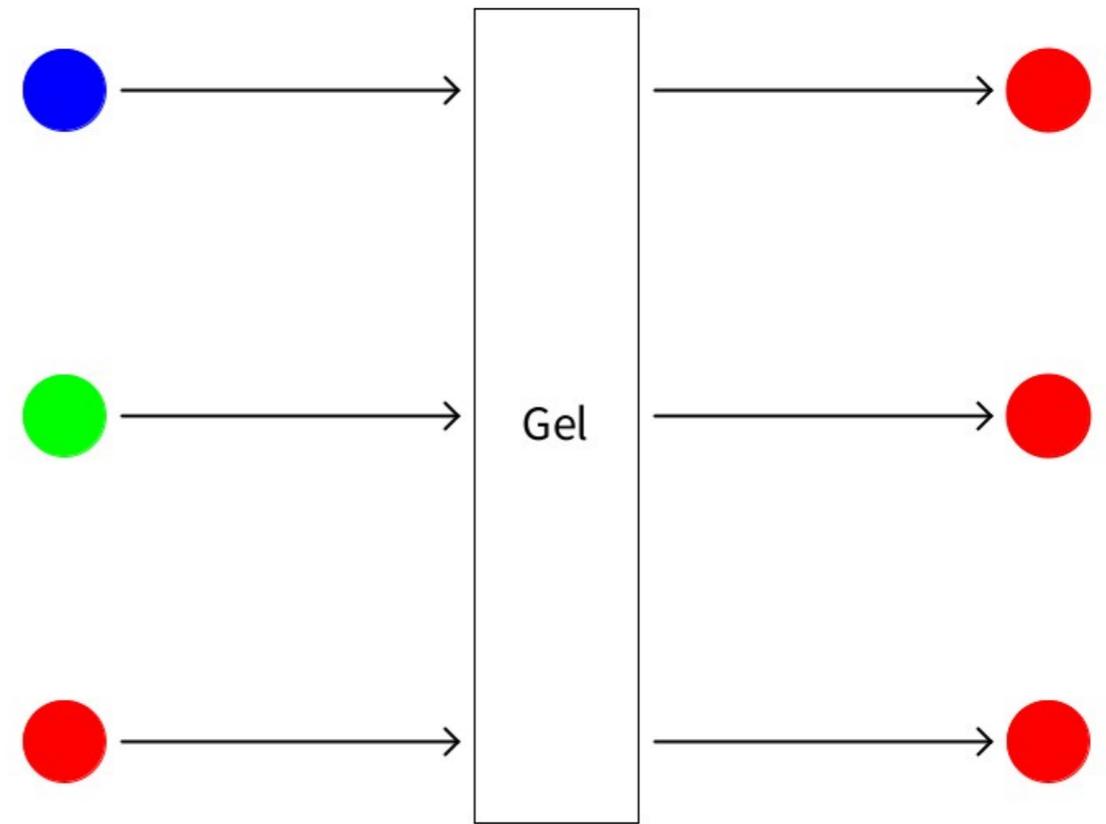
An Example of the Scientific Method



The RGB Model - Two Theories for Paddles (Gels)

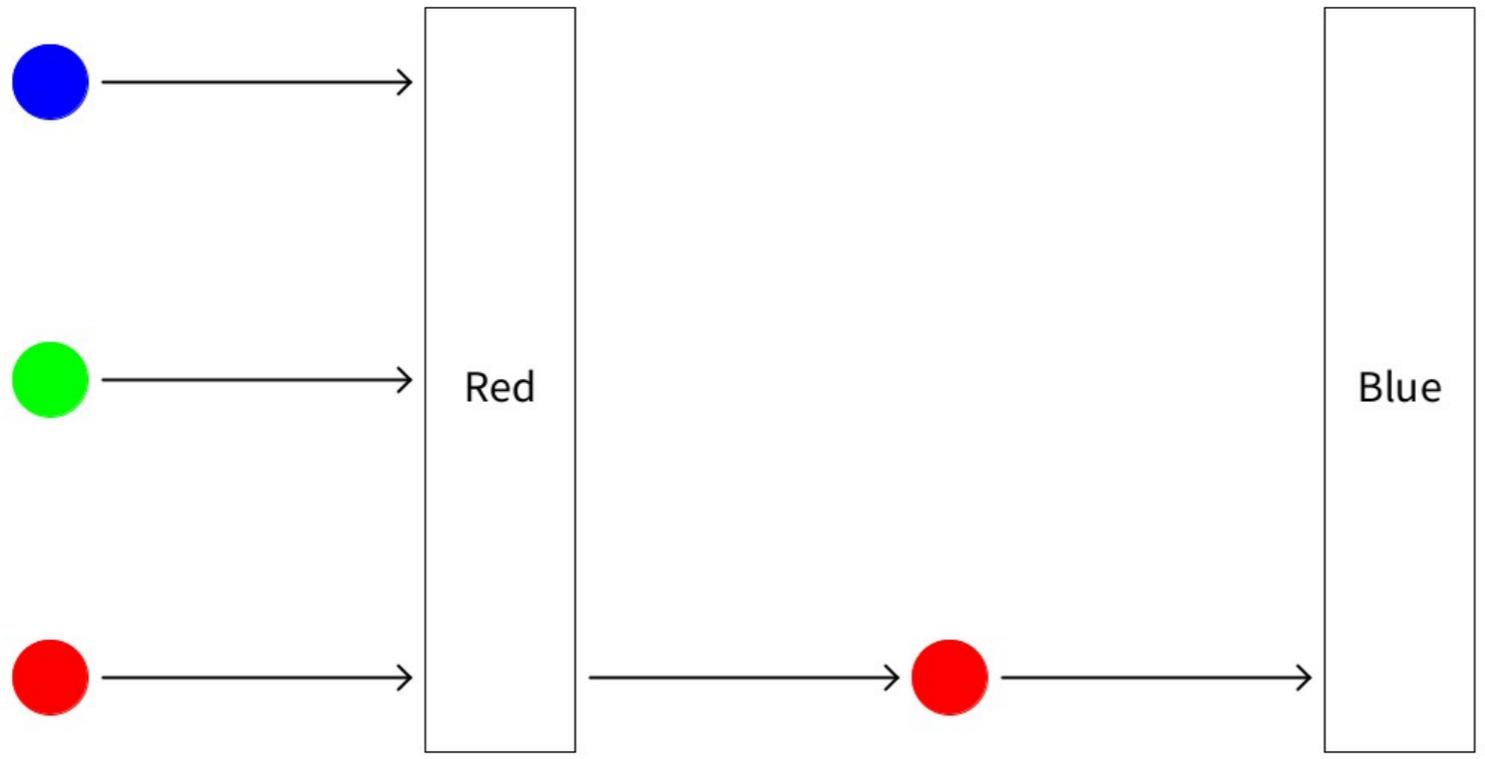


Block (Filter)

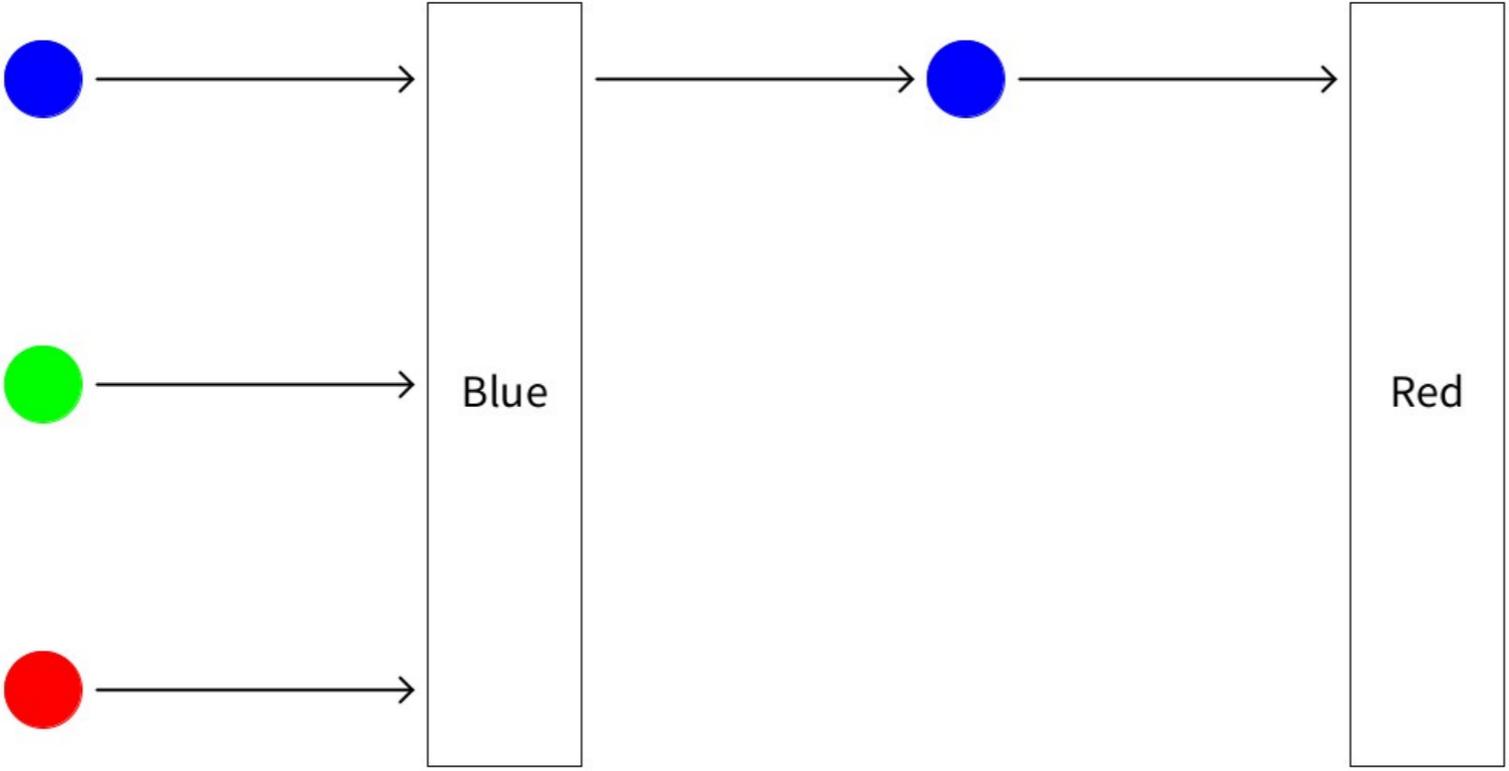


Paint (Change)

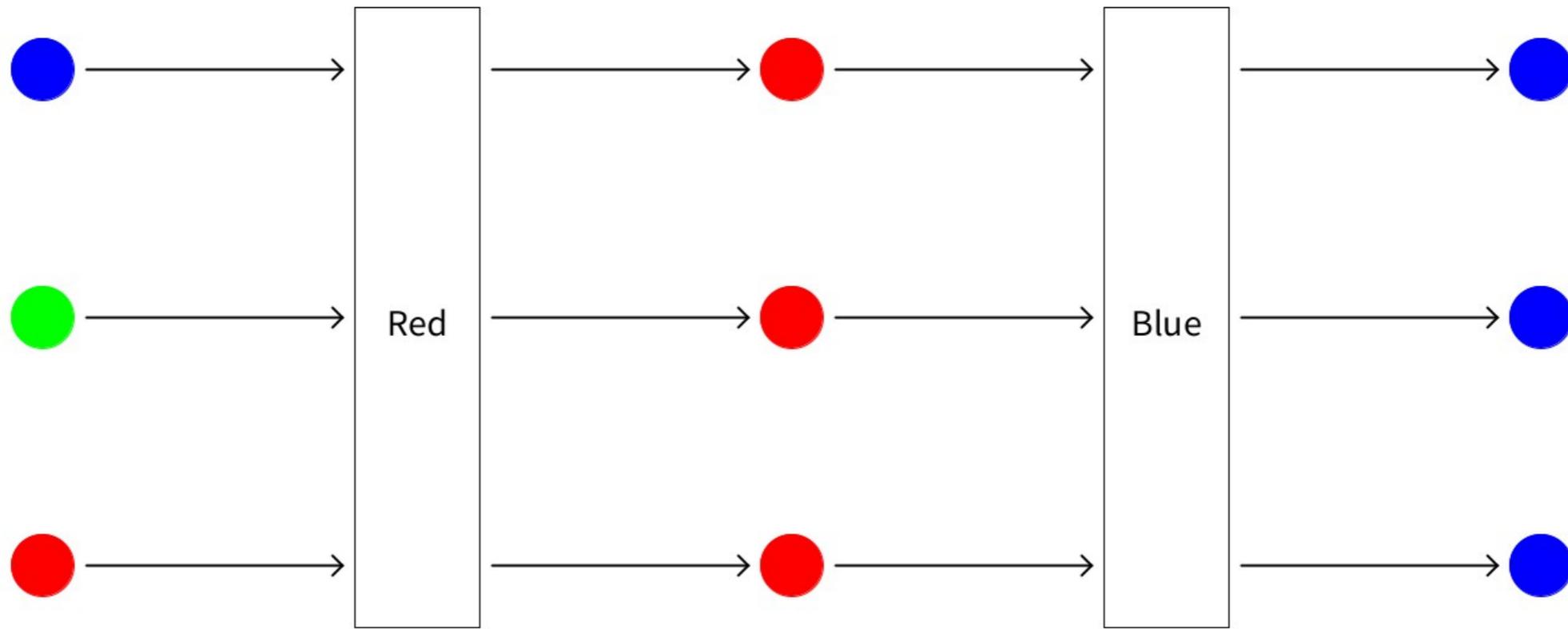
Design an Experiment to choose between the two theories.



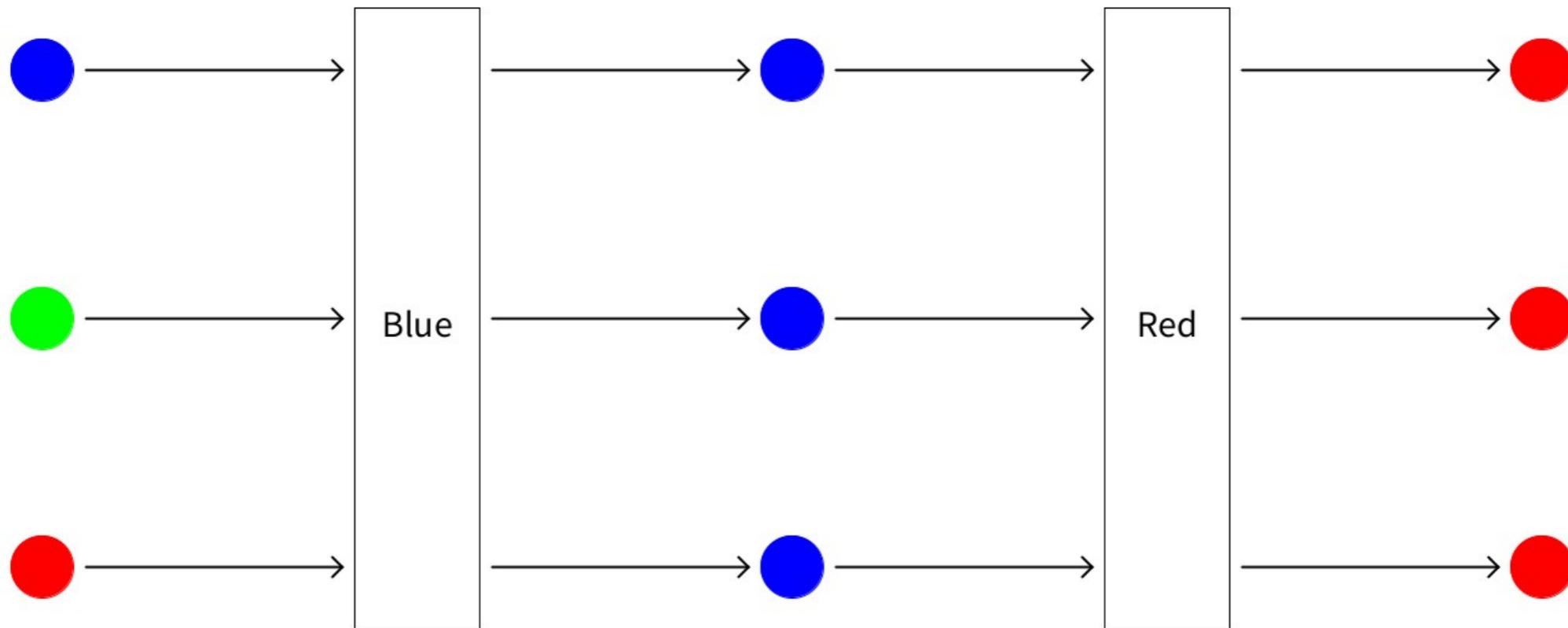
Blocking model -- first red gel then blue gel



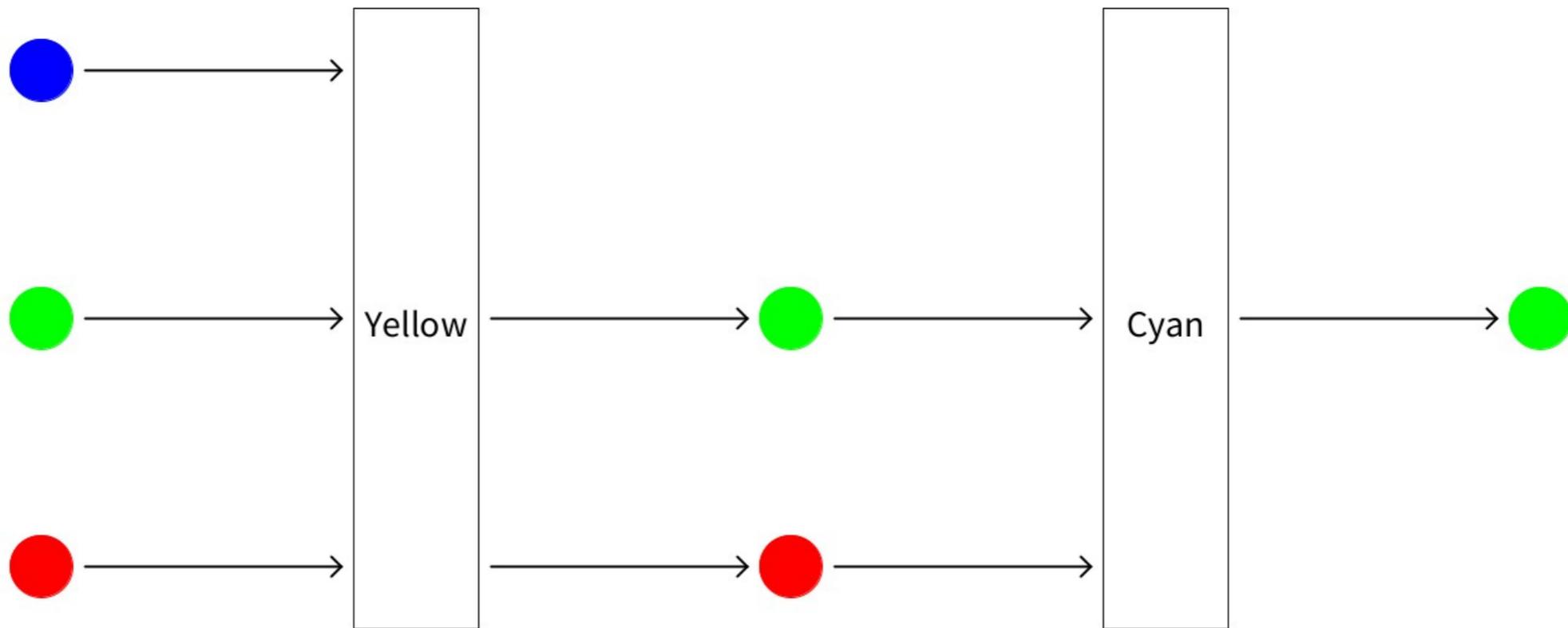
Blocking model -- first blue gel then red gel



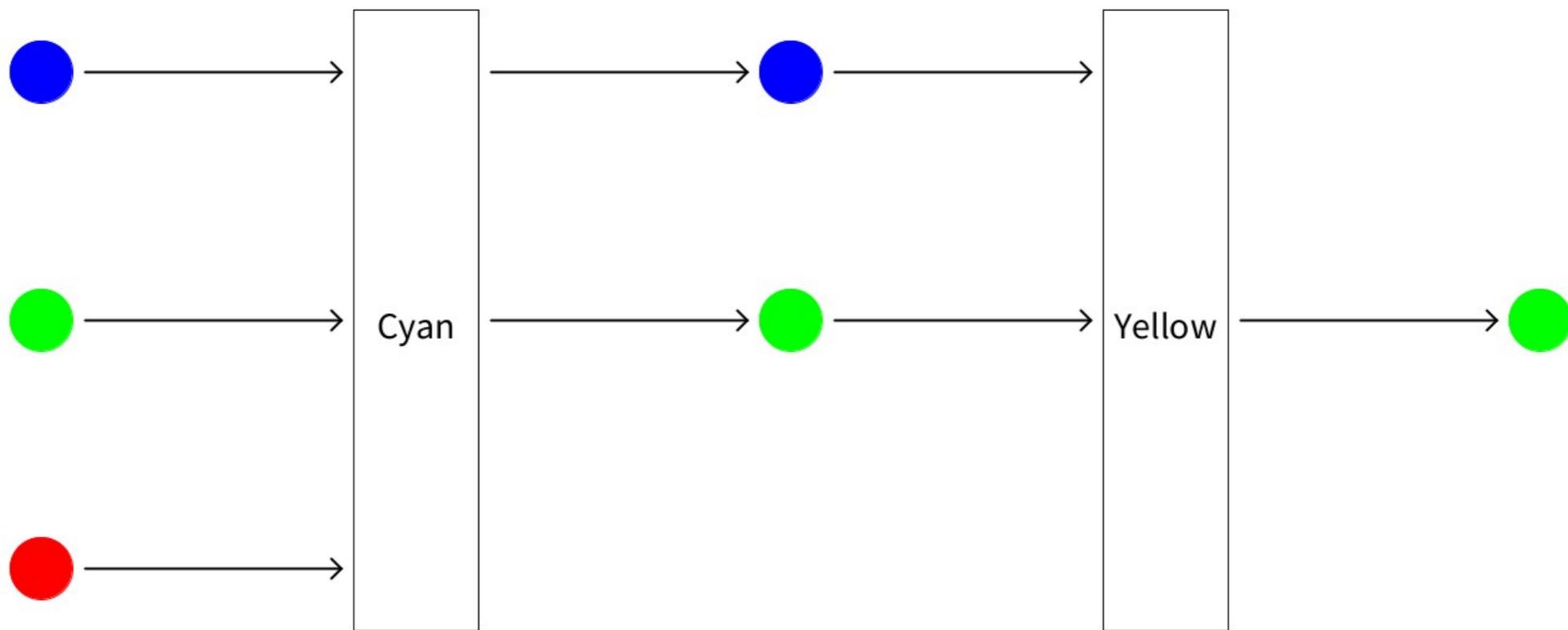
Paint model -- first red gel then blue gel



Paint model -- first blue gel then red gel



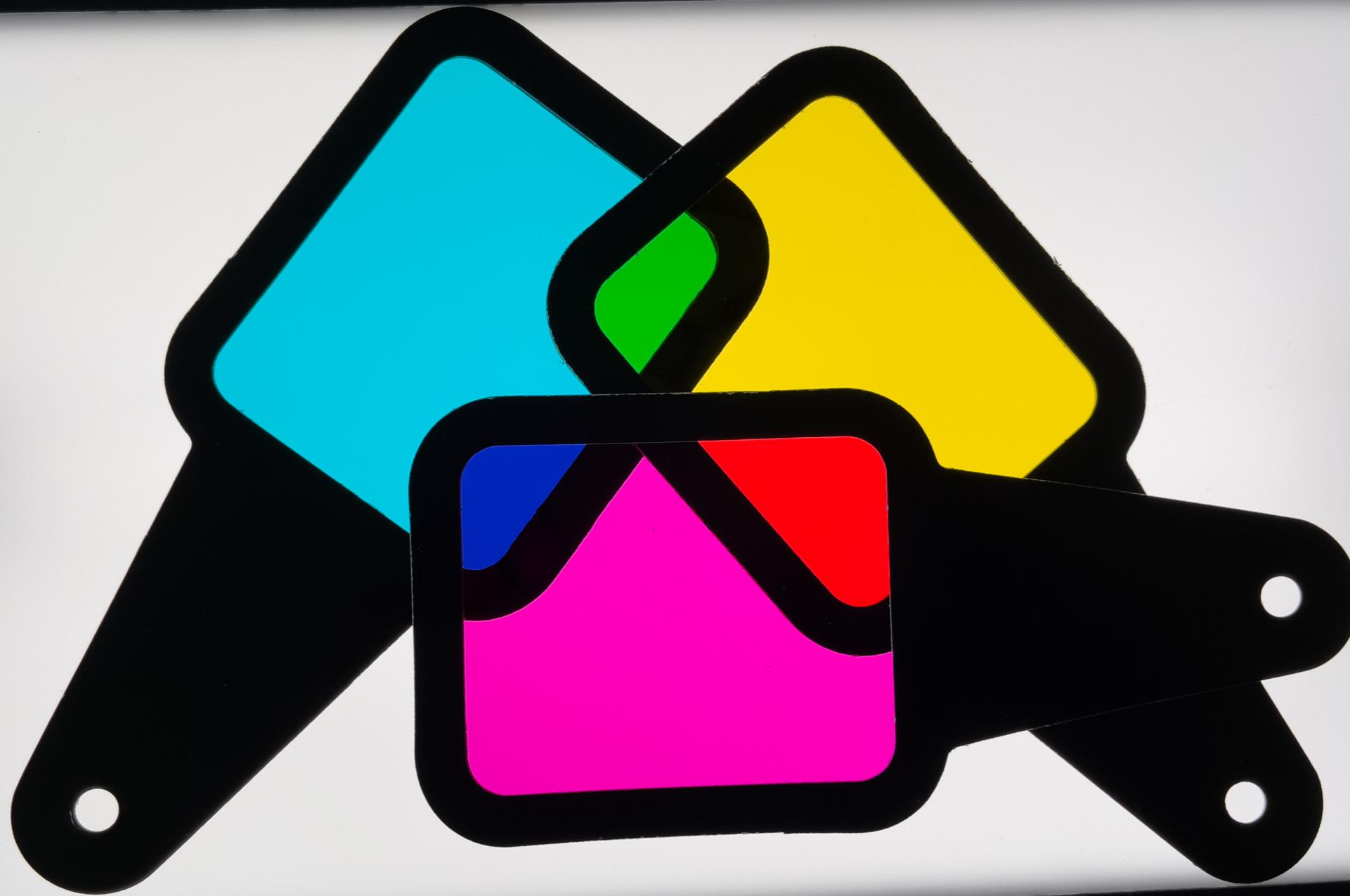
Blocking model -- first yellow gel then cyan gel



Blocking model -- first cyan gel then yellow gel

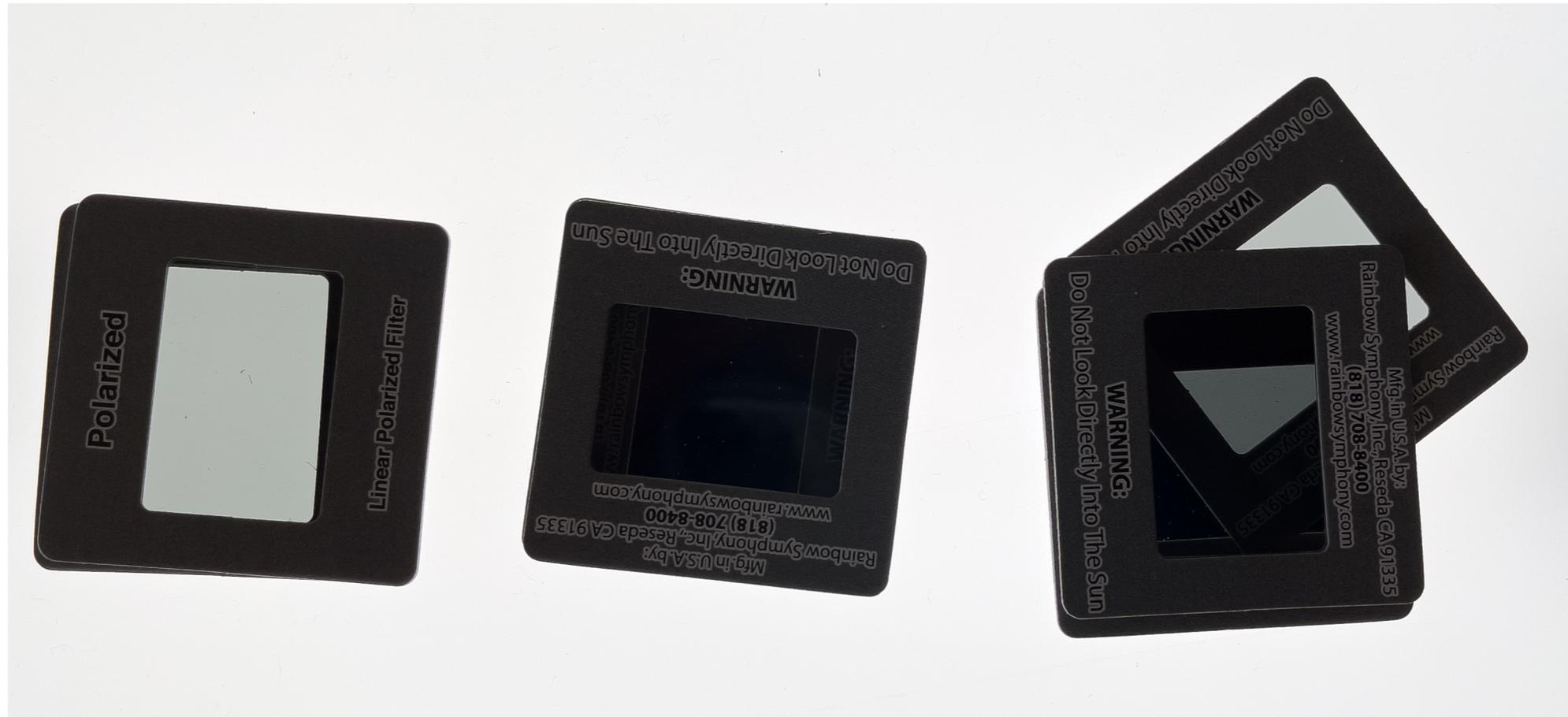


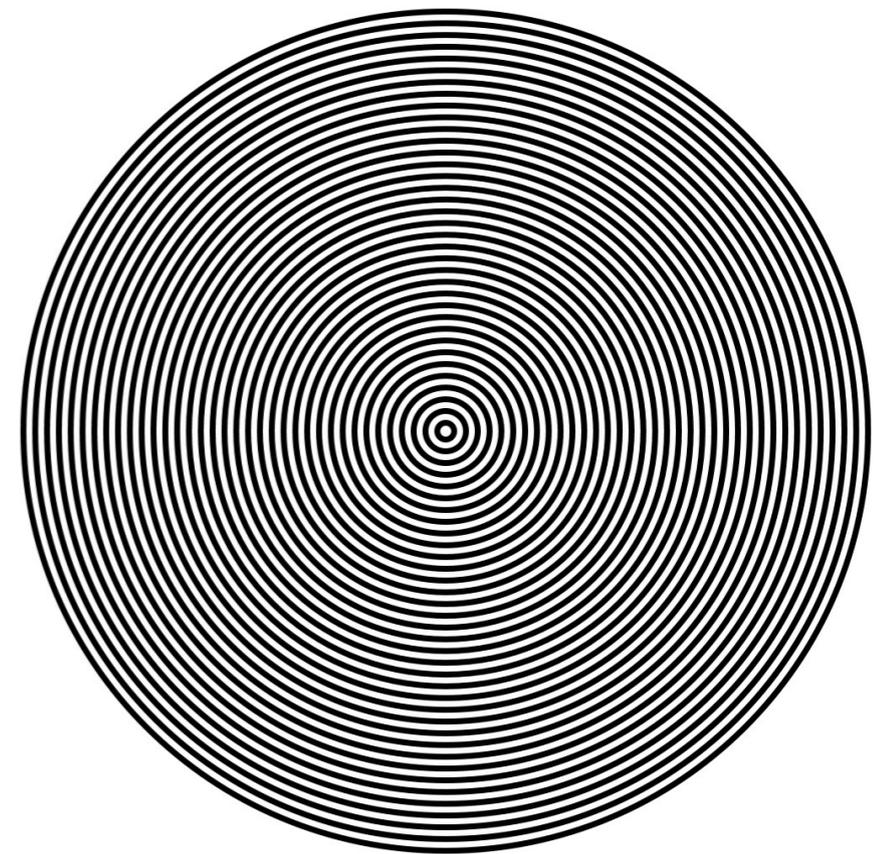
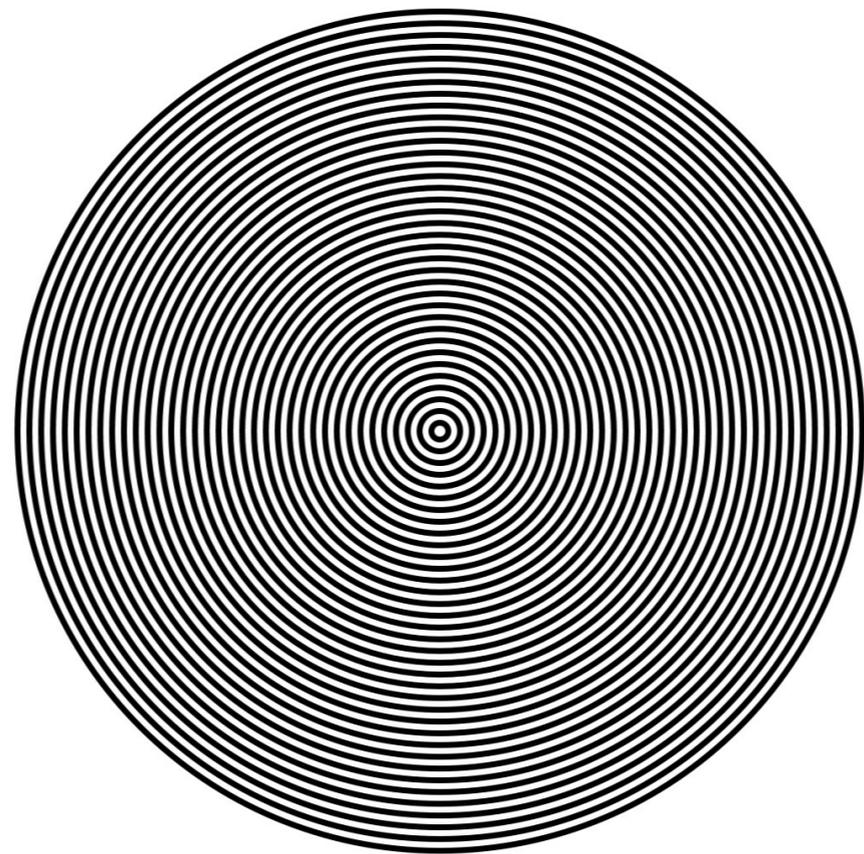
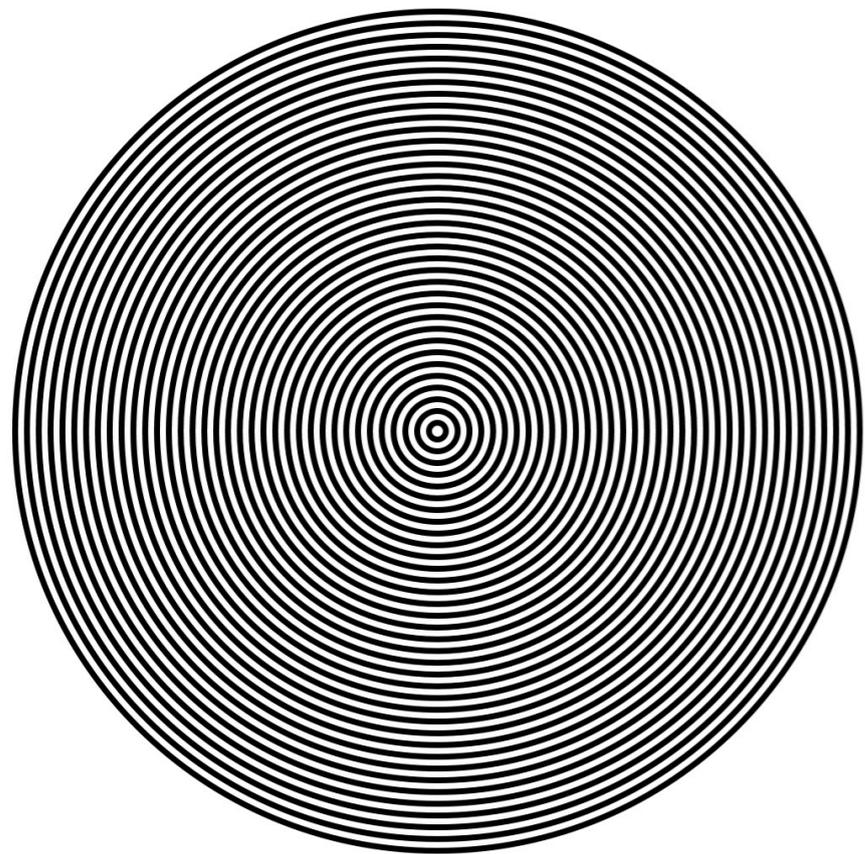
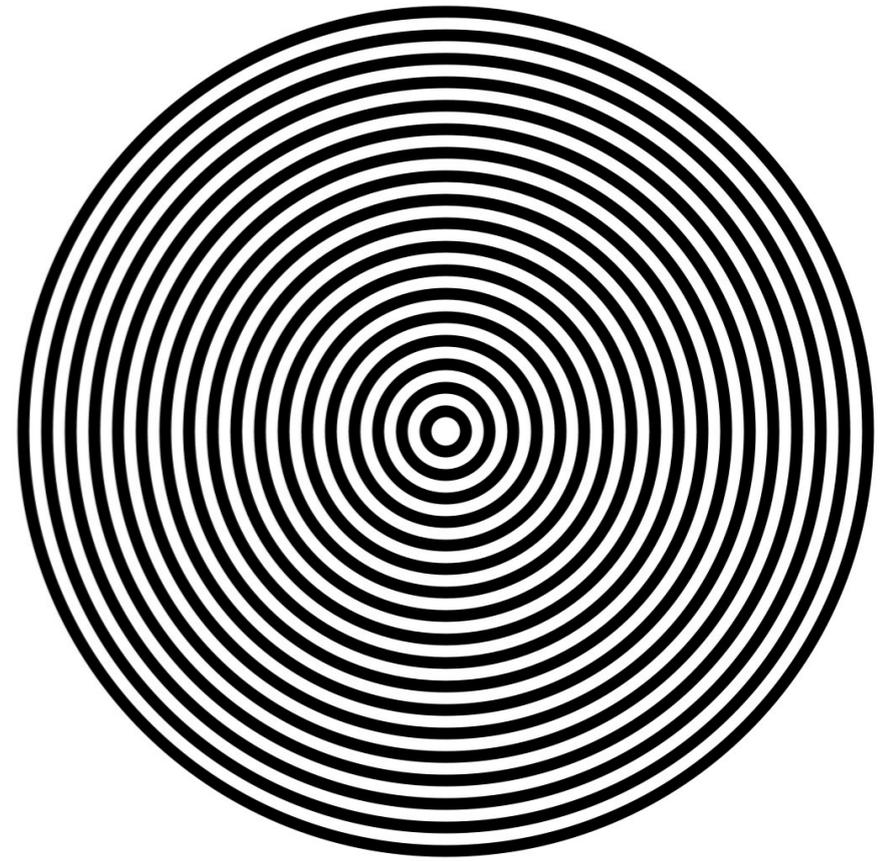
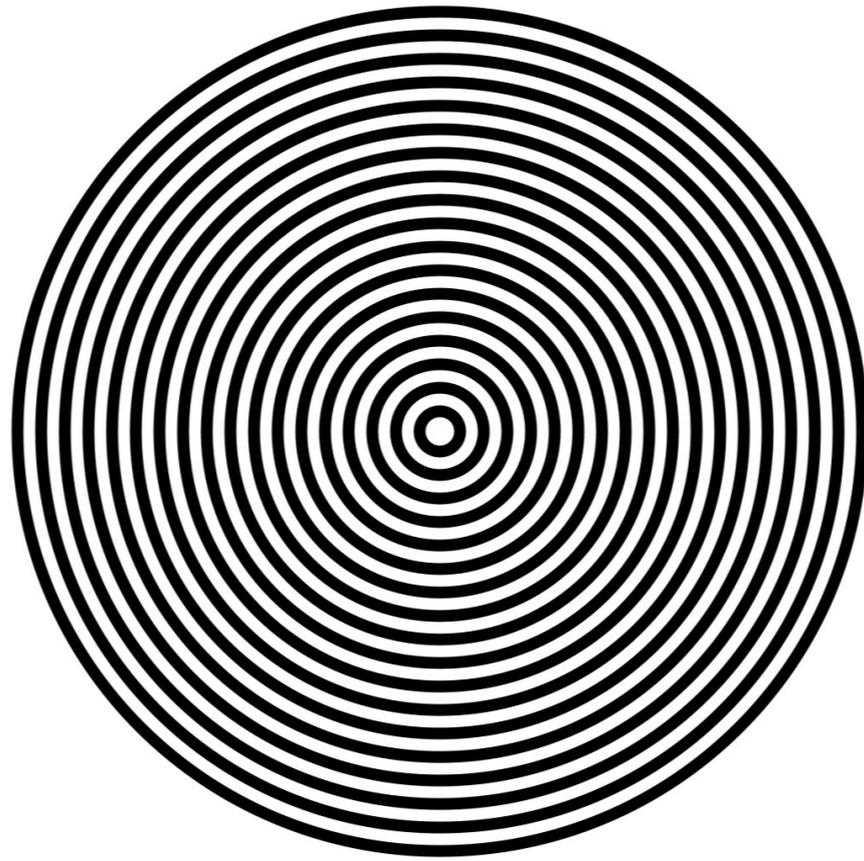
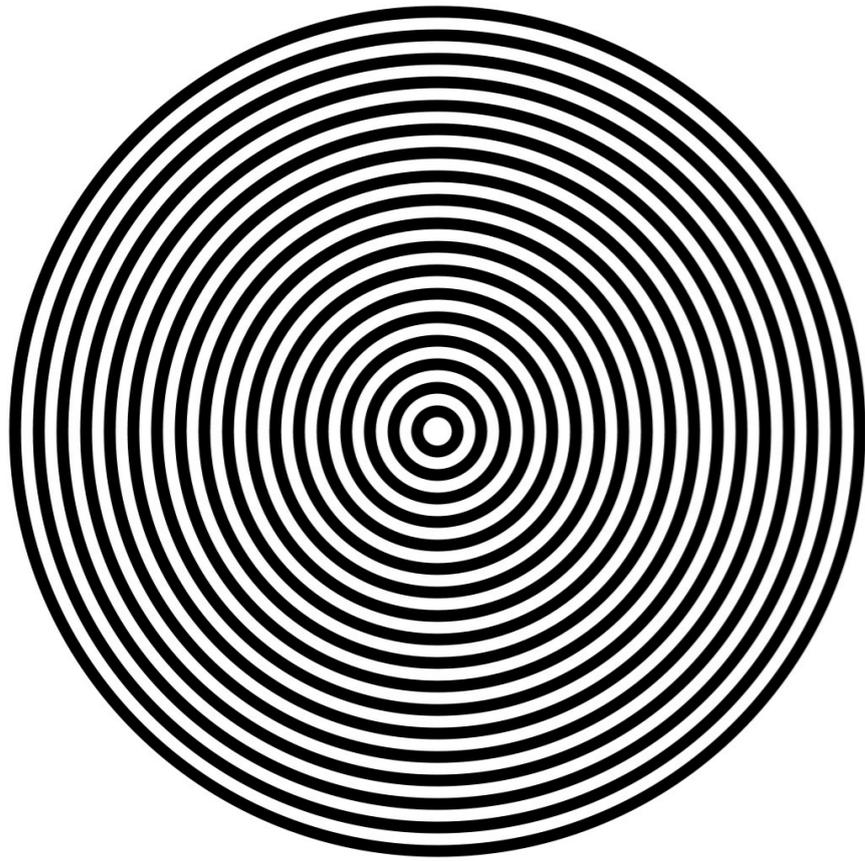
Kodak
film



Kodak
film

Polarized Light

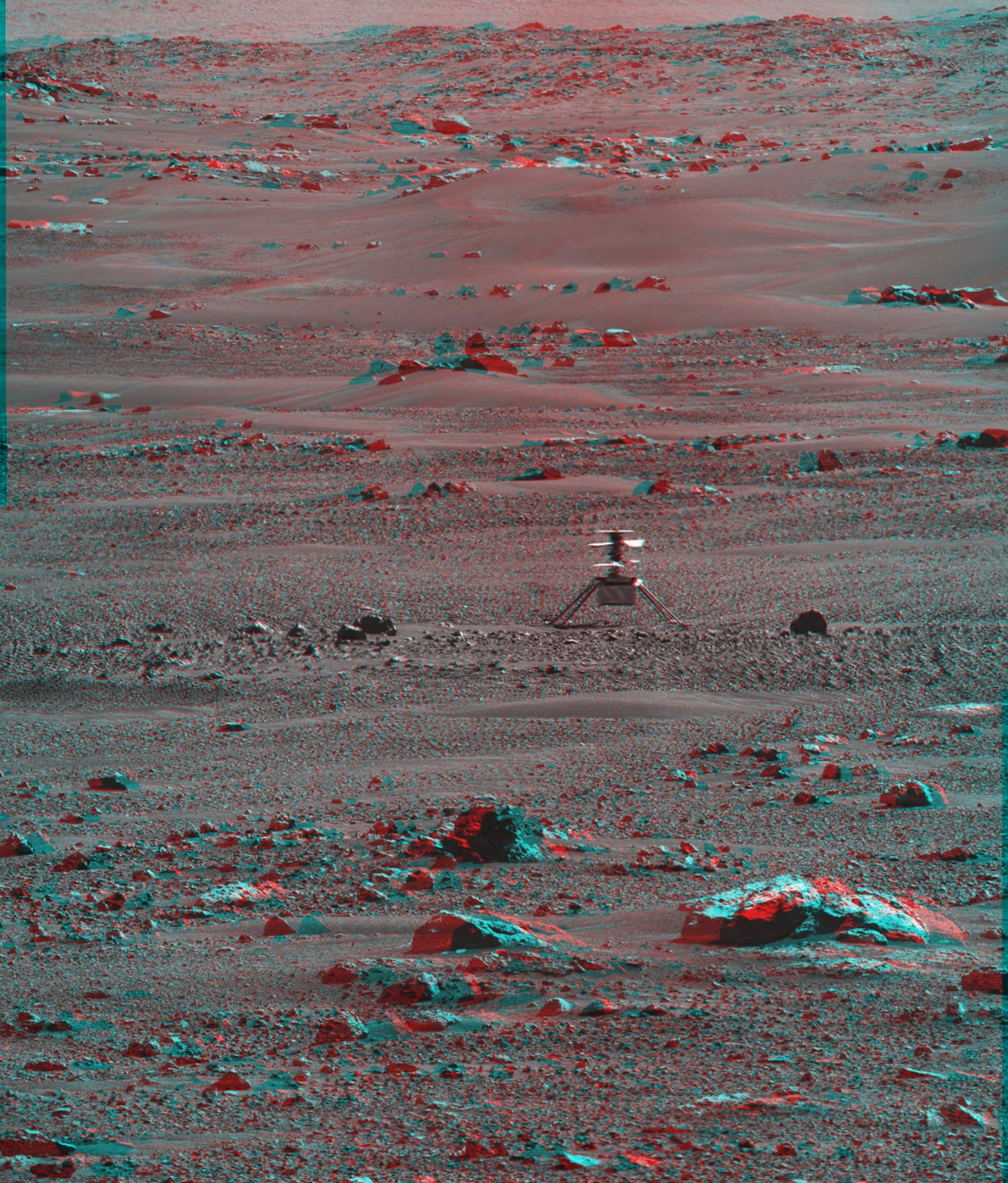




Descriptive vs. Explanatory

Too Many Parameters -- Falsifiable?
(Lee Smolin, *The Trouble With Physics*)

Machine Learning vs Human Learning



Cultural Cognition

Dan Kahan

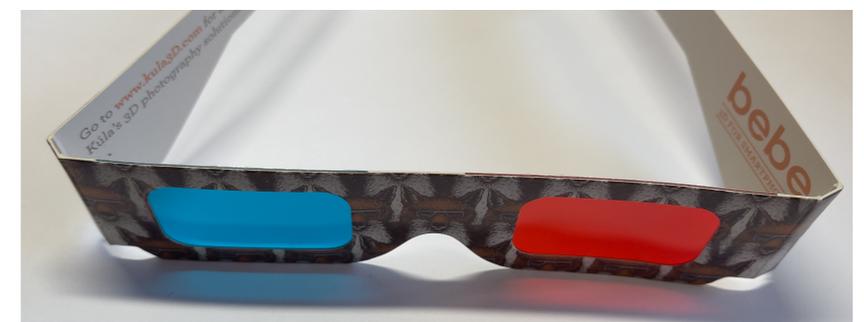
Seeing and Thinking,

Fast and Slow

Daniel Kahneman

Modeling and
Decision-Making

in High Stakes,
Often Controversial
Settings





SIGN IN

Have I Been Trained?

🔍 Enter text or upload an image...



This website:

I understand

- uses cookies to enhance the user experience
- uses a NSFW filter, but should be treated as NSFW
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A New Element -- AI/ML driven actors that pass the Turing test
(but so did George Santos)

- ChatGPT
- Text-based image generation
- Text-based or image based image search -- <https://haveibeentrained.com/>

customer service
automate routine conversations

profit

Service workers

Customers

lower 'tense' conversations

care-giving

Writers

Artists

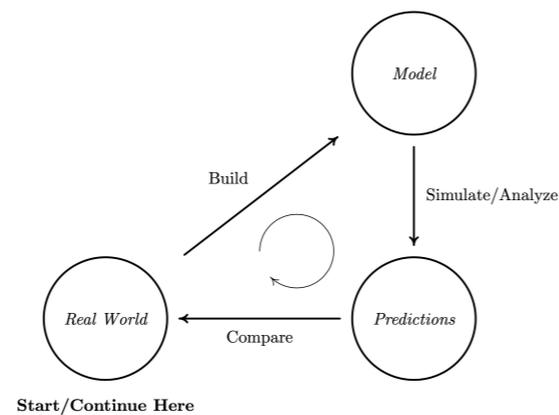
pre-screening for live agents

customer understanding

Executives

Stockholders

faster service



un-biased decisions

People who need care

Health care workers

Tech workers

Extraverts

targeted marketing

customer acquisition

Introverts

Present population

more 'social' interactions

patient listener

Future population

product improvement

Modeling, Goals and Stakeholders (AI/ML)

Are We Bits or Qubits?

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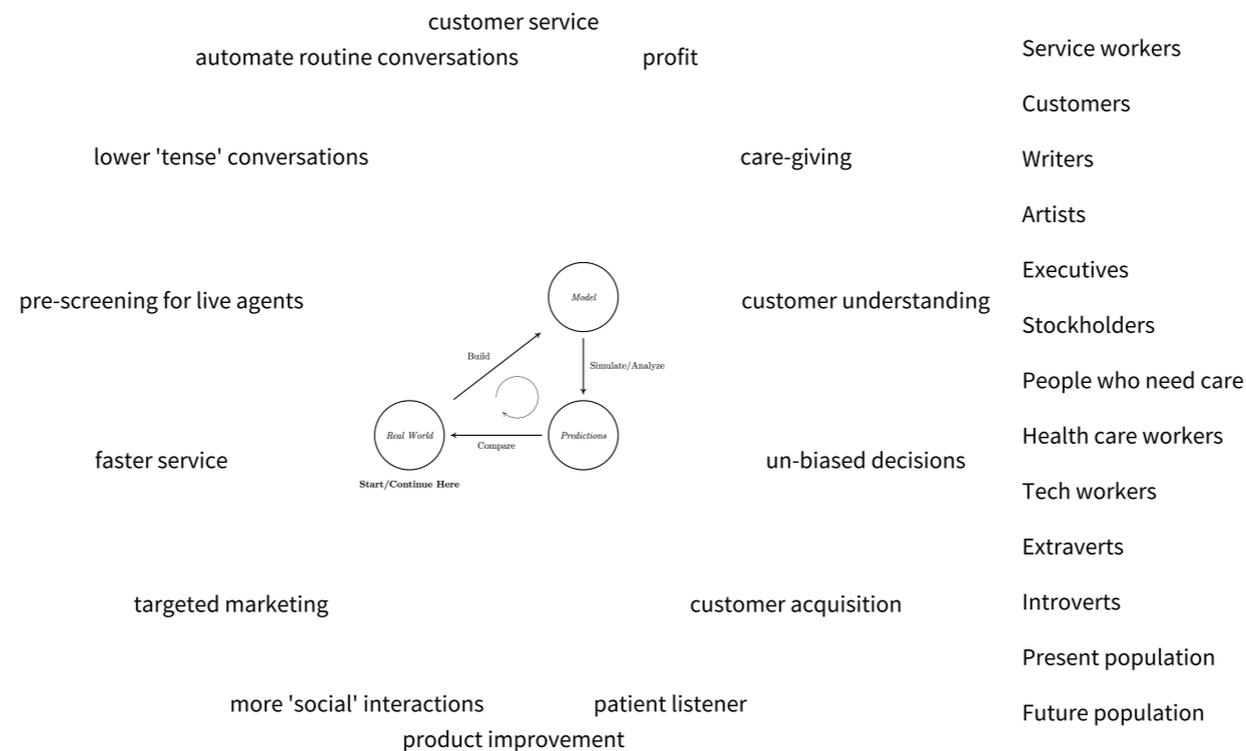
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