Flat Mirrors and Geometry

Figure 1: Flat Mirrors at Hessian Lake in Bear Mountain State Park

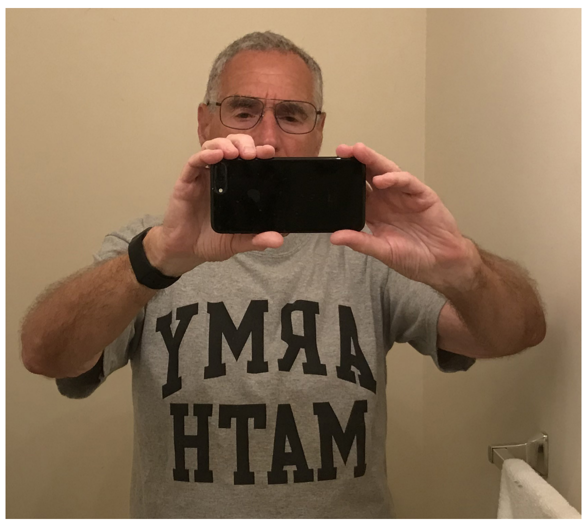

Figure 2: A Flat Bathroom Mirror

Flat mirrors are everywhere and the "mirror images" they produce are easily understood with a bit of elementary geometry – lines, angles and triangles. The two flat mirrors in Figure 1 are at Hessian Lake in New York State's Bear Mountain State Park but you'll find flat mirrors in puddles after it rains and in your bathroom. See Figure 2. Notice the familiar "mirror-image" – the writing on the author's shirt is backwards – reversed from left-to-right.

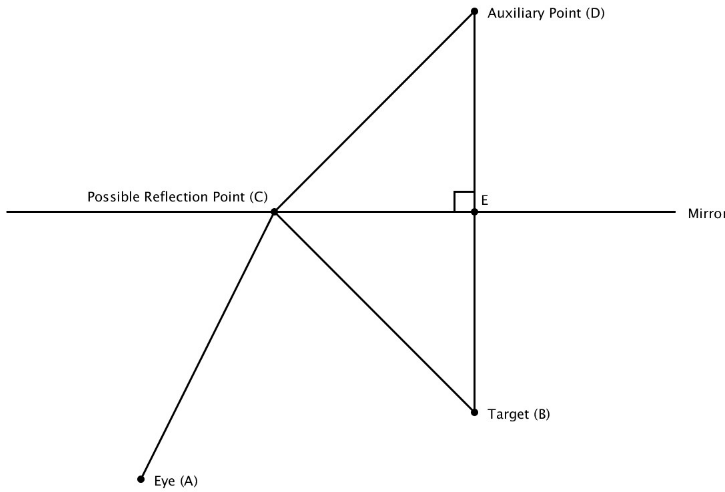

Figure 3: A Possible path

You may wonder why the mirror image is reversed left-to-right instead of upside-down. If so, you're not alone. If you Google "Why do mirrors flip left ..." or "Why do mirrors reverse left ..." the autocomplete will complete your question and provide links to many web pages that address this question. We'll begin with a simple experiment that helps answer this question. Start with a thin sheet of paper – one that you can see through. Write the words "Army Math" on the paper and hold it on your chest as you look in the mirror. As you continue looking at those words in the mirror extend your arms straight in front of you so that you can see through the paper. Compare the words in the mirror with the words you see looking through the paper.

We begin our study of reflection in mirrors with Fermat's Principle. In its first and simplest form Fermat's Principle is almost anthropomorphic. It says that light rays traveling between two points follow the fastest path.

Figure 3 shows a mirror, an eye (at the point A) and a target (at the point B). Of course the shortest path from the target to the eye is just a straight line but we are interested in more complicated paths – paths that go from the target to the mirror, hitting the mirror at a point C, and then traveling to the eye. These paths are made up of two straight line segments, the line segment BC and the line segment AC. The light rays that travel on this kind of path will pick the reflection point C to minimize the total length of the two segments BC and AC. We must figure out where on the mirror this best reflection point is.

Still looking at Figure 3 we draw a line that is perpendicular to the mirror and goes through the point B. Then we mark an auxiliary point D on this line that is the same distance from the mirror as the point B but behind the mirror. Notice that the two triangles ΔCED and ΔCEB are congruent. Therefore the line segments CD and CB have the same length. Therefore the path – from B to C and then from C to A has the same total length as the path from D to C and then from C to A. If we choose the point C to be on the line from A to D as shown in Figure 4 then the path from A to C and then to D is the shortest possible path and, thus, so is the path from A to C and then to B.

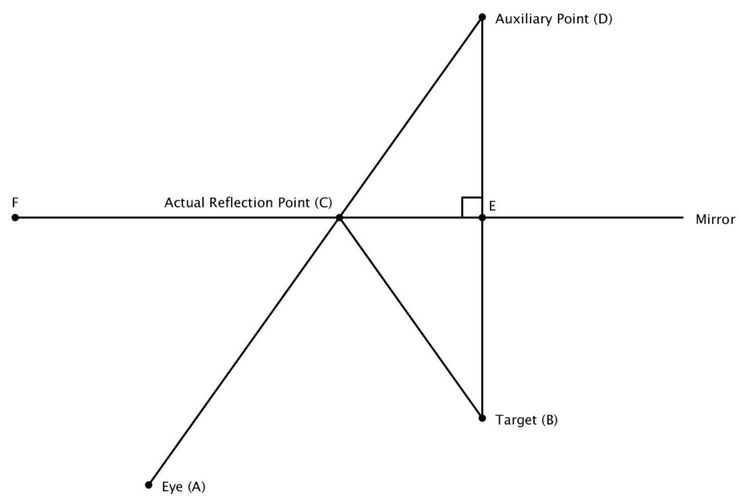

Figure 4: The fastest (shortest) path

Notice the angle $\angle FCA$ is the same as the angle $\angle DCE$. The angle $\angle FCA$ is the angle at which the light ray bounces off the mirror. Notice the angle $\angle BCE$ is the same as the angle $\angle DCE$ because the two triangles ΔCED and ΔCEB are congruent are congruent. The angle $\angle BCE$ is the angle at which the light ray from the target to the mirror hits the mirror.

We sense depth – that is, we build three-dimensional models of our world – using binocular vision. In Figure 5 we add a second eye and do the same analysis we did for the first eye. Our brain takes the input from the two eyes and assumes that the incoming light rays are straight lines without any changes of direction. These two straight lines are called **apparent paths**. So our brain assumes that the target is at the point where the two apparent paths meet – that is, at the auxiliary point D. Those the apparent position of the target is behind the mirror directly opposite its actual position and the same distance behind the mirror as the actual target is in front of the mirror. Mirror images are reversed left-to-right because they are directly behind the mirror directly opposite actual images. In effect, we are looking at the actual image from behind.

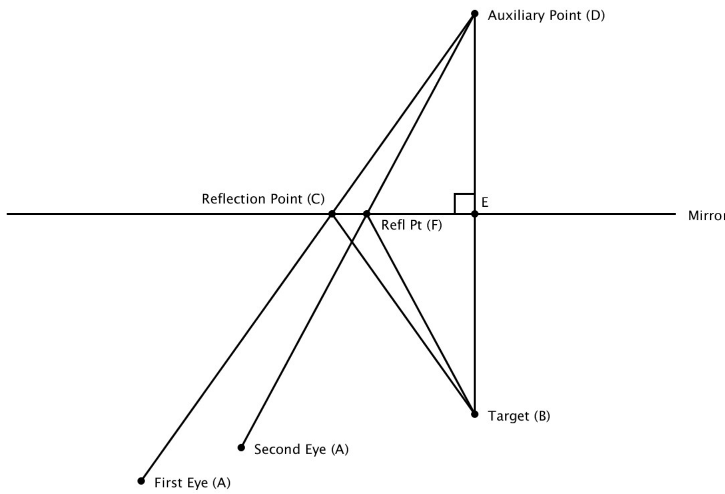

Figure 5: Two eyes and binocular vision